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# Neutralinos and Higgs Bosons in the Next-To-Minimal Supersymmetric Standard Model

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**Abstract**

The purpose of this paper is to present a complete and consistent list of the Feynman rules for the vertices of neutralinos and Higgs bosons in the Next-To-Minimal Supersymmetric Standard Model (NMSSM), which does not yet exist in the literature. The Feynman rules are derived from the full expression for the Lagrangian and the mass matrices of the neutralinos and Higgs bosons in the NMSSM. Some crucial differences between the vertex functions of the NMSSM and the Minimal Supersymmetric Standard Model (MSSM) are discussed.

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# 1 Introduction

With the observation of the top quark [1] the particle content of the electroweak Standard Model (SM) [2] is completely experimentally confirmed with the exception of the Higgs sector [3]. There merely exists an upper bound for the Higgs mass $m_\Phi$ of about 1 TeV due to unitarity constraints [4] and a lower bound of about 65 GeV from the so far unsuccessful experimental Higgs search [5].

However, in the SM there are some unsolved problems connected with this minimal Higgs sector. First, the coupling constants do not meet at one point at high energies in the simplest Grand Unified Theory (GUT) extensions of the SM [6]. Also experimental bounds for the proton decay impose severe constraints on non-supersymmetric GUTs. Further, the SM does not explain the small ratio between the energy scale of the electroweak symmetry breaking and the Planck scale ($m_W^2/m_P^2 \approx 10^{-34}$) [7] (hierarchy problem) and does not answer the question how the large radiative corrections of the order of the GUT scale to the Higgs mass are prevented [8] (naturalness or fine tuning problem).

Supersymmetric models may provide solutions to these problems without abandoning the idea of Higgs bosons as elementary particles. The boson-fermion symmetry stabilizes the Higgs mass against radiative corrections, electroweak symmetry breaking can be triggered at the required scale and unification of the coupling constants at a single point is possible. The simplest supersymmetric model, the Minimal Supersymmetric Standard Model (MSSM) [9], is characterized by a minimal particle content, explicit supersymmetry breaking by soft symmetry breaking terms and an exact symmetry called $R$ parity, which guarantees conservation of baryon and lepton number. In the MSSM two Higgs doublet fields $H_1$ and $H_2$ with vacuum expectation values $v_1$ and $v_2$ ($\tan\beta = v_2/v_1$) are needed in order to avoid anomalies and to give masses to both up-type and down-type quarks. They lead to five physical Higgs bosons, two neutral scalar, one neutral pseudoscalar, and a pair of charged Higgs particles.

The MSSM predicts for every particle of the SM a partner with a spin differing by 1/2, called gaugino, higgsino, scalar lepton (slepton) and scalar quark (squark), respectively. Since "ordinary" and supersymmetric particles obviously have different masses, supersymmetry must be broken in nature. In most phenomenological models, this is simulated by the introduction of explicit soft supersymmetry breaking terms in the Lagrangian which split the masses within a supermultiplet. Moreover, the soft supersymmetry breaking terms lead to the formation of new mass eigenstates in the supersymmetric sector. Photinos, zinos and the neutral higgsinos form neutralinos as mass eigenstates, the mass eigenstates composed of winos and charged higgsinos are the charginos, and also the left-handed and right-handed scalar quarks and leptons mix to new eigenstates.

But the constraint $\rho \equiv m_W^2/(m_Z^2 \cos^2\theta_W) \approx 1$ also allows extended supersymmetric models with additional Higgs doublets, singlets or even triplets. Such nonminimal supersymmetric models gain more and more attraction since they can evade some of the constraints of the MSSM and may lead to a variety of new phenomena with interesting experimental signatures.

In this paper we focus on the Next-To-Minimal Supersymmetric Standard Model (NMSSM) [10, 11], the minimal extension of the MSSM by a gauge singlet superfield. The Higgs sector of the NMSSM contains five physical neutral Higgs bosons, three Higgs

scalars $S_a$ ($a = 1, 2, 3$) and two pseudoscalars $P_\alpha$ ($\alpha = 1, 2$), and two degenerate physical charged Higgs particles $C^\pm$. The neutralino sector is extended to five neutralinos instead of four in the MSSM, with masses and eigenstates determined by a $5 \times 5$ mixing matrix. The remaining particle content is identical with that of the MSSM.

The NMSSM was first developed within the framework of GUTs and superstring theories [12, 13, 14]. It is mainly motivated by its potential to eliminate the $\mu$ problem of the MSSM [15], where the origin of the the $\mu$ parameter in the superpotential

$$W_{\mathrm{MSSM}} = \mu H_1 H_2 \tag{1}$$

is not understood. For phenomenological reasons it has to be of the order of the electroweak scale, while the "natural" mass scale would be of the order of the GUT or Planck scale. This problem is evaded in the NMSSM where the $\mu$ term in the superpotential is dynamically generated through $\mu = \lambda x$ with a dimensionless coupling $\lambda$ and the vacuum expectation value $x$ of the Higgs singlet.

As another essential feature of the NMSSM the mass bounds for the Higgs bosons and neutralinos are weakened. While in the MSSM experimental data imply lower mass bounds of 18 GeV for the lightest neutralino [16] (assuming either $\tan\beta > 2$ or the gluino mass $m_{\tilde{g}} > 100$ GeV), 44 GeV for the lightest scalar and 21 GeV for the lightest pseudoscalar Higgs boson [17] very light or massless neutralinos and Higgs bosons are not excluded in the NMSSM [18, 19]. Furthermore the upper tree level mass bound for the lightest Higgs scalar of the MSSM

$$m_h^2 \leq m_Z^2 \cos^2 2\beta \tag{2}$$

is increased to

$$m_{S_1}^2 \leq m_Z^2 \cos^2 2\beta + \lambda^2 (v_1^2 + v_2^2) \sin^2 2\beta \,. \tag{3}$$

Both bounds are raised by radiative corrections by about 30 GeV [20, 21, 22, 23]. Taking into account the weak coupling of a Higgs scalar of singlet type the NMSSM may still remain a viable model when the MSSM can be ruled out due to eq. (2).

The above arguments make an intensive study of the NMSSM very desirable. While the implications for supersymmetric phenomenology have already been studied to some extent [22, 24, 25, 26], one finds, however, only incomplete lists of Feynman rules for the NMSSM in the literature [11, 27, 28]. Furthermore, different sign conventions for the parameters in the superpotential have been established [21, 22, 25]. In this paper we provide the full Lagrangian of the NMSSM and present a complete list of all Feynman rules for the neutral Higgs bosons and neutralinos which differ from those of the MSSM. These differences between NMSSM and MSSM may arise in two manners: The singlet component of a Higgs boson or neutralino can explicitly appear in the vertex factor, or the Feynman rules of NMSSM and MSSM are formally equal differing just by the mixing of the Higgs bosons or neutralinos.

The production of Higgs bosons or neutralinos is among the most promising processes suitable for the discovering of minimal or nonminimal supersymmetric signatures. Unfortunately, until now, no supersymmetric particles have been observed. The experimental results at the high energy colliders can be transformed into mass bounds for the supersymmetric particles and excluded domains in the parameter space. We use the derived

vertex factors in order to review our previous analysis of the allowed parameter space [18, 19] in the NMSSM exploring the meanwhile improved experimental limits from LEP. For LEP2 and a future linear collider, neutralino and Higgs production in the NMSSM have been studied in refs. [25, 29, 30].

Comparing the Higgs couplings of NMSSM and MSSM we point out some fundamental differences between these models. A crucial test for supersymmetry as well as for the standard model would be the measurement of the Higgs self-coupling, e. g. at double Higgs production at the NLC [31, 32]. Further significant differences could arise in Higgs production via gluon fusion or Higgs decays into photons, gluons, neutralinos or charginos. We emphasize that we do not want to discuss explicit supersymmetric processes but provide all necessary Feynman rules for their computation. A detailed study of cross sections and decay rates would exceed the intention of this paper by far. This has been either done in other works or remains as a future challenge.

The outline of this paper is as follows: First we describe in Sec. 2 the complete Lagrangian of the NMSSM including all terms for the self-interaction of the gauge multiplets, the interaction of gauge and matter multiplets as well as the self-interaction of the matter multiplets. Explicit expressions for the scalar potential and the soft supersymmetry breaking potential are given. Since the additional singlet superfield of the NMSSM leads to extended Higgs and neutralino sectors, we present the Higgs and neutralino mixings and review also the chargino and slepton/squark mixings in order to fix all conventions and to show the influence of all parameters of the model. The main part of this paper (Sec. 3) is dedicated to the Feynman rules of the NMSSM which are derived from the relevant parts of the Lagrangian. In Secs. 4 and 5 we illustrate the differences between the MSSM and NMSSM couplings. Sec. 4 contains a discussion of the Higgs couplings to gauge bosons and an analysis of the experimental constraints on the parameter space and the Higgs and neutralino masses due to the results at the high energy colliders. In Sec. 5 we compare in detail the Higgs couplings to quarks, scalar quarks, neutralinos and charginos and the trilinear Higgs self-couplings of NMSSM and MSSM and indicate the phenomenological implications for supersymmetric processes to be studied in future works.

# 2 The Lagrangian of the NMSSM

The NMSSM is characterized by its superpotential

$$\begin{aligned} W &= \lambda \varepsilon_{ij} H_1^i H_2^j N - \frac{1}{3} k N^3 \\ &\quad + h_u \varepsilon_{ij} \tilde{Q}^i \tilde{U} H_2^j - h_d \varepsilon_{ij} \tilde{Q}^i \tilde{D} H_1^j - h_e \varepsilon_{ij} \tilde{L}^i \tilde{R} H_1^j \end{aligned} \tag{4}$$

where $H_1 = (H_1^0, H^-)$ and $H_2 = (H^+, H_2^0)$ are the $SU(2)$ Higgs doublets with hypercharge $-1/2$ and $1/2$ and vacuum expectation values $(v_1, 0)$, $(0, v_2)$ ($\tan\beta = v_2/v_1$), respectively, $N$ is the Higgs singlet with hypercharge 0 and vacuum expectation value $x$, and $\varepsilon_{ij}$ is totally antisymmetric with $\varepsilon_{12} = -\varepsilon_{21} = 1$. The notation of the squark/slepton doublets and singlets is conventional, generation indices are understood. Contrary to the MSSM, the superpotential of the NMSSM consists only of trilinear terms with dimensionless couplings. Therefore all terms of the unbroken theory are scale-invariant with the

supersymmetry breaking scale $m_{\rm SUSY}$ as the only mass scale, while the MSSM contains as additional mass scale the parameter $\mu$ which leads to the above described $\mu$ problem.

The scalar potential

$$V = \frac{1}{2}(D^a D^a + D'^2) + F_i^* F_i, \tag{5}$$

is composed of the $D$ and $F$ terms

$$\begin{aligned} D^a &= g A_i^* T_{ij}^a A_j \,, & (6)\\ D' &= \frac{1}{2} g' y_i A_i^* A_i \,, & (7)\\ F_i &= \partial W / \partial A_i \,, & (8) \end{aligned}$$

with the generators $(T_{ij}^a)$ of the weak isospin group $SU(2)$ and the hypercharge $y_i$. The $A_i$ are the scalar fields of the theory.

We now give the explicit expressions of the $D$ and $F$ terms as a function of doublet and singlet fields and the couplings $\lambda$, $k$, $h_u$, $h_d$ and $h_e$:

$$\begin{aligned} V_D &= \frac{1}{2}(D^a D^a + D'^2) \\ &= \frac{1}{8} g^2 \Big[ (H_1^{i*} H_1^i)^2 + (H_2^{i*} H_2^i)^2 + (\tilde{Q}^{i*} \tilde{Q}^i)^2 + (\tilde{L}^{i*} \tilde{L}^i)^2 \\ &\quad +4|H_1^{i*} H_2^i|^2 - 2(H_1^{i*} H_1^i)(H_2^{j*} H_2^j) + 4|H_1^{i*} \tilde{Q}^i|^2 - 2(H_1^{i*} H_1^i)(\tilde{Q}^{j*} \tilde{Q}^j) \\ &\quad +4|H_1^{i*} \tilde{L}^i|^2 - 2(H_1^{i*} H_1^i)(\tilde{L}^{j*} \tilde{L}^j) + 4|H_2^{i*} \tilde{Q}^i|^2 - 2(H_2^{i*} H_2^i)(\tilde{Q}^{j*} \tilde{Q}^j) \\ &\quad +4|H_2^{i*} \tilde{L}^i|^2 - 2(H_2^{i*} H_2^i)(\tilde{L}^{j*} \tilde{L}^j) + 4|\tilde{Q}^{i*} \tilde{L}^i|^2 - 2(\tilde{Q}^{i*} \tilde{Q}^i)(\tilde{L}^{j*} \tilde{L}^j) \Big] \\ &\quad + \frac{1}{8} g'^2 \Big[ H_2^{2*} H_2^2 + H_2^{1*} H_2^1 - H_1^{1*} H_1^1 - H_1^{2*} H_1^2 \\ &\quad + y_Q (\tilde{Q}^{1*} \tilde{Q}^1 + \tilde{Q}^{2*} \tilde{Q}^2) + y_u \tilde{U}^* \tilde{U} + y_d \tilde{D}^* \tilde{D} \\ &\quad - \tilde{L}^{1*} \tilde{L}^1 - \tilde{L}^{2*} \tilde{L}^2 + 2 \tilde{R}^* \tilde{R} \Big]^2, \end{aligned} \tag{9}$$

$$\begin{aligned} V_F &= F_i^* F_i \\ &= |\lambda \varepsilon_{ij} H_1^i H_2^j - k N^2|^2 \\ &\quad + |\lambda H_2^2 N + h_d \tilde{Q}^2 \tilde{D} + h_e \tilde{L}^2 \tilde{R}|^2 + |\lambda H_2^1 N + h_d \tilde{Q}^1 \tilde{D} + h_e \tilde{L}^1 \tilde{R}|^2 \\ &\quad + |\lambda H_1^2 N + h_u \tilde{Q}^2 \tilde{U}|^2 + |\lambda H_1^1 N + h_u \tilde{Q}^1 \tilde{U}|^2 \\ &\quad + |h_u H_2^2 \tilde{U} - h_d H_1^2 \tilde{D}|^2 + |-h_u H_2^1 \tilde{U} + h_d H_1^1 \tilde{D}|^2 \\ &\quad + |h_u \varepsilon_{ij} \tilde{Q}^i H_2^j|^2 + |h_d \varepsilon_{ij} \tilde{Q}^i H_1^j|^2 + |h_e \varepsilon_{ij} \tilde{L}^i H_1^j|^2 \\ &\quad + |h_e H_1^2 \tilde{R}|^2 + |h_e H_1^1 \tilde{R}|^2. \end{aligned} \tag{10}$$

The Yukawa interactions and fermion mass terms arise by the following part of the Lagrangian:

$$\mathcal{L}_{\rm Yukawa} = -\frac{1}{2}[(\partial^2 W / \partial A_i \partial A_j) \psi_i \psi_j + {\rm h.c.}] \,, \tag{11}$$

where the two-component spinors $\psi_i$ are the supersymmetric partners of the scalar fields $A_i$.

Since supersymmetric particles have not been found in the low-energy particle spectrum there must exist a mechanism (if SUSY is realized at all) that breaks SUSY and splits the masses of the different members of a supermultiplet. In the NMSSM as well as in the MSSM one simulates the supersymmetry breaking by adding explicit soft supersymmetry breaking terms to the Lagrangian. The most general supersymmetry breaking potential [33] can be written as

$$\begin{aligned} V_{\text{soft}} &= m_1^2|H_1|^2 + m_2^2|H_2|^2 + m_3^2|N|^2 \\ &+ m_Q^2|\tilde{Q}|^2 + m_U^2|\tilde{U}|^2 + m_D^2|\tilde{D}|^2 \\ &+ m_L^2|\tilde{L}|^2 + m_E^2|\tilde{R}|^2 \\ &-(\lambda A_\lambda \varepsilon_{ij} H_1^i H_2^j N + \text{h.c.}) - (\frac{1}{3} k A_k N^3 + \text{h.c.}) \\ &+(h_u A_U \varepsilon_{ij} \tilde{Q}^i \tilde{U} H_2^j - h_d A_D \varepsilon_{ij} \tilde{Q}^i \tilde{D} H_1^j - h_e A_E \varepsilon_{ij} \tilde{L}^i \tilde{R} H_1^j + \text{h.c.}) \\ &+\frac{1}{2} M \lambda^a \lambda^a + \frac{1}{2} M' \lambda' \lambda'. \end{aligned} \tag{12}$$

The scalar potential as well as the soft supersymmetry breaking potential appear with negative signs in the Lagrangian of the NMSSM.

Finally, we give for completeness the parts of the Lagrangian responsible for the self-interaction of the gauge multiplet and for the interaction of gauge and matter multiplets. The gauge field interacts with itself and the gauginos via

$$\begin{aligned} \mathcal{L}_V &= -\frac{1}{4} V_{\mu\nu}^a V^{a\mu\nu} - \frac{1}{4}(\partial_\mu V_\nu' - \partial_\nu V_\mu')^2 \\ &\quad -i\bar{\lambda}^a(\delta_{ab}\sigma^\mu \partial_\mu - f_{abc}\sigma^\mu V_\mu^c)\lambda^b, \end{aligned} \tag{13}$$

where

$$V_{\mu\nu}^a = \partial_\mu V_\nu^a - \partial_\nu V_\mu^a + g f_{abc} V_\mu^a V_\nu^a \tag{14}$$

and $f_{abc}$ are the structure constants of the nonabelian gauge group $SU(2)$. The Pauli matrices are denoted by $\sigma^\mu = (1, \vec{\sigma})$. The interaction between gauge and matter multiplets is described by

$$\begin{aligned} \mathcal{L}_M &= (-g T_{ij}^a V_\mu^a - \frac{1}{2} g' y_i \delta_{ij} V_\mu')(\bar{\psi}_i \bar{\sigma}^\mu \psi_j + i A_i^* \overleftrightarrow{\partial}^\mu A_j) \\ &\quad + ig\sqrt{2} T_{ij}^a (\lambda^a \psi_j A_i^* - \bar{\lambda}^a \bar{\psi}_i A_j) + \frac{ig'}{\sqrt{2}} y_i (\lambda' \psi_i A_i^* - \bar{\lambda}' \bar{\psi}_i A_i) \\ &\quad + A_i^* A_j (g T_{ik}^a V_\mu^a + \frac{1}{2} g' y_i \delta_{ik} V_\mu')(g T_{kj}^b V^{\mu b} + \frac{1}{2} g' y_j \delta_{kj} V'^\mu) \end{aligned} \tag{15}$$

Since the additional Higgs singlet field has hypercharge 0, it does not interact with gauge and matter fields. Therefore, this part of the Lagrangian is unchanged compared to the MSSM. In eqs. (13) – (15), $V_\mu^a$ and $V_\mu'$ denote the $SU(2)$ and $U(1)$ gauge fields of the model, respectively, $\lambda^a$ and $\lambda'$ are their gaugino fermionic partners.

Now the interaction Lagrangian of the NMSSM is complete. As free parameters appear the ratio of the doublet vacuum expectation values, $\tan\beta$, the singlet vacuum expectation value $x$, the couplings in the superpotential $\lambda$ and $k$, the parameters $A_\lambda$, $A_k$, as well as $A_U$,

$A_D$, $A_E$ (for all three generations) in the supersymmetry breaking potential, the gaugino mass parameters $M$ and $M'$, and the scalar mass parameters for the Higgs bosons $m_{1,2,3}$, squarks $m_{Q,U,D}$ and sleptons $m_{L,E}$. Note that the sign convention for the gaugino mass parameters in the NMSSM is normally chosen to be opposite to that of the MSSM in order to recover the minimal model with $\mu = \lambda x$ in the limit $x \to \infty$, with $\lambda x, kx$ fixed.

This low-energy Lagrangian of the NMSSM obviously contains more free parameters than the MSSM, the couplings $\lambda$ and $k$ as well as the parameters $A_\lambda$ and $A_k$ are absent in the minimal model. Within the framework of supergravity theories, however, the total number of parameters remains the same in both models. At the GUT scale, the couplings $\lambda$ and $k$ have to be fixed in the NMSSM instead of $\mu$ and $B$ (the coefficient of the quadratic Higgs term in the soft supersymmetry breaking potential) in the MSSM. Then all other parameters at the electroweak scale follow by renormalization group equations [34]. The mass spectrum of such a constrained NMSSM has been studied in refs. [21, 35, 36].

Even without imposing unification constraints there exist some restrictions for the low-energy parameters: Explicit CP violation in the scalar sector is avoided by choosing the parameters $\lambda$, $A_\lambda$, $k$ and $A_k$ to be real. Further, a sufficient condition for the vacuum to conserve CP is to assume $\lambda$ and $k$ to be positive which allows a choice of vacuum with $v_1, v_2, x > 0$ [11].

In the next step we will study the mixing in the Higgs, neutralino/chargino and slepton/squark sector before deriving the Feynman rules.

## 2.1 The Higgs sector

The $10\times10$ Higgs mass squared matrix decouples in two $3\times3$ blocks for the CP-even scalar and CP-odd pseudoscalar Higgs bosons, respectively, and two $2\times2$ blocks for the charged Higgs particles. One eigenvalue of the CP-odd and charged Higgs matrices vanishes and corresponds to an unphysical Goldstone mode. The minimization conditions for the scalar potential $\partial V/\partial v_{1,2} = 0$, $\partial V/\partial x = 0$ eliminate three parameters of the Higgs sector which are normally chosen to be $m_1^2$, $m_2^2$ and $m_3^2$. Then at tree level the elements of the symmetric CP-even mass squared matrix $\mathcal{M}_S^2 = (M_{ij}^{S\,2})$ become in the basis $(H_1, H_2, N)$

$$
\begin{aligned}
M_{11}^{S\,2} &= \frac{1}{2}v_1^2({g'}^2 + g^2) + \lambda x \tan\beta(A_\lambda + kx), && (16)\\
M_{12}^{S\,2} &= -\lambda x(A_\lambda + kx) + v_1 v_2(2\lambda^2 - \frac{1}{2}{g'}^2 - \frac{1}{2}g^2) && (17)\\
M_{13}^{S\,2} &= 2\lambda^2 v_1 x - 2\lambda k x v_2 - \lambda A_\lambda v_2, && (18)\\
M_{22}^{S\,2} &= \frac{1}{2}v_2^2({g'}^2 + g^2) + \lambda x \cot\beta(A_\lambda + kx), && (19)\\
M_{23}^{S\,2} &= 2\lambda^2 v_2 x - 2\lambda k x v_1 - \lambda A_\lambda v_1, && (20)\\
M_{33}^{S\,2} &= 4k^2x^2 - kA_k x + \frac{\lambda A_\lambda v_1 v_2}{x}. && (21)
\end{aligned}
$$

In the same way one finds for the elements of the CP-odd matrix $\mathcal{M}_P^2$

$$
M_{11}^{P\,2} = \lambda x(A_\lambda + kx)\tan\beta, \qquad (22)
$$

$$
\begin{aligned}
M_{12}^{P\,2} &= \lambda x(A_\lambda + kx), && (23)\\
M_{13}^{P\,2} &= \lambda v_2(A_\lambda - 2kx), && (24)\\
M_{22}^{P\,2} &= \lambda x(A_\lambda + kx)\cot\beta, && (25)\\
M_{23}^{P\,2} &= \lambda v_1(A_\lambda - 2kx), && (26)\\
M_{33}^{P\,2} &= \lambda A_\lambda \frac{v_1 v_2}{x} + 4\lambda k v_1 v_2 + 3kA_k x, && (27)
\end{aligned}
$$

and for the charged Higgs matrix one obtains

$$
\mathcal{M}_c^2 = \left(\lambda A_\lambda x + \lambda k x^2 - v_1 v_2 \left(\lambda - \frac{g^2}{2}\right)\right) \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix}. \qquad (28)
$$

All Higgs mass matrices obtain radiative corrections from loops of the heavy quarks, scalar quarks, Higgs particles, higgsinos, gauge bosons and gauginos. Appropriate formulae can be found in refs. [21, 22, 23].

Assuming CP conservation in the Higgs sector, the Higgs matrices are diagonalized by the real orthogonal $3 \times 3$ matrices $U^S$ and $U^P$, respectively,

$$
\begin{aligned}
\mathrm{Diag}(m_{S_1}^2, m_{S_2}^2, m_{S_3}^2) &= U^{S\,T} \mathcal{M}_S^2 U^S, && (29)\\
\mathrm{Diag}(m_{P_1}^2, m_{P_2}^2, 0) &= U^{P\,T} \mathcal{M}_P^2 U^P, && (30)
\end{aligned}
$$

where $m_{S_1} < m_{S_2} < m_{S_3}$ and $m_{P_1} < m_{P_2}$ denote the Higgs masses in ascending order. The mass eigenstates $S_a$ $(a = 1, 2, 3)$ of the neutral scalar Higgs bosons, $P_\alpha$ $(\alpha = 1, 2)$ of the physical neutral pseudoscalar Higgs particles and $C^\pm$ of the physical charged Higgs boson are obtained by the transformations

$$
\begin{aligned}
\begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix} &= \sqrt{2} U^S \left[ \begin{pmatrix} \mathrm{Re} H_1^0 \\ \mathrm{Re} H_2^0 \\ \mathrm{Re} N \end{pmatrix} - \begin{pmatrix} v_1 \\ v_2 \\ x \end{pmatrix} \right], && (31)\\
\begin{pmatrix} P_1 \\ P_2 \\ P_G \end{pmatrix} &= \sqrt{2} U^P \begin{pmatrix} \mathrm{Im} H_1^0 \\ \mathrm{Im} H_2^0 \\ \mathrm{Im} N \end{pmatrix}, && (32)\\
C^+ &= \cos\beta H_2^1 + \sin\beta H_1^{2*}. && (33)
\end{aligned}
$$

Note that only the upper $3 \times 2$ matrix of $U^P$ is physically relevant since the eigenstate $P_G$ of the CP-odd matrix corresponds to an unphysical Goldstone mode. Since the diagonalization matrices are normally to be found numerically, we do not bother about analytical results. For the rest of this paper, indices of the scalar Higgs bosons are denoted by latin characters which can take values from 1 to 3 while greek letters with possible values 1 or 2 are used for the pseudoscalar Higgs bosons.

The phenomenology of the NMSSM Higgs sector has been studied in ref. [22]. At tree level, the masses and mixings of the Higgs bosons depend on six parameters: the couplings in the superpotential $\lambda$ and $k$, the ratio of the vacuum expectation values of the Higgs doublets $\tan\beta$, the vacuum expectation value $x$ of the singlet and the parameters in the supersymmetry breaking potential $A_\lambda$ and $A_k$. In addition, the radiative corrections are

influenced by the squark masses and $A$-terms in the supersymmetry breaking potential as well as by the parameters of the gaugino/higgsino sector.

For comparison with the MSSM we also quote the corresponding results for the Higgs sector of the minimal supersymmetric model. In the MSSM there exist two neutral scalar Higgs bosons $h$ and $H$ ($m_h < m_H$), one physical neutral pseudoscalar Higgs particle $A$ and a pair of degenerate charged Higgs bosons. Their masses and mixings are conventionally parameterized in terms of $m_A$ and $\tan\beta$. Then the tree level mass squared matrix of the Higgs scalars can be written as

$$\mathcal{M}^2 = \begin{pmatrix} m_Z^2 \cos^2\beta + m_A^2 \sin^2\beta & -(m_Z^2 + m_A^2)\sin\beta\cos\beta \\ -(m_Z^2 + m_A^2)\sin\beta\cos\beta & m_Z^2 \sin 2\beta + m_A^2 \cos^2\beta \end{pmatrix}. \tag{34}$$

The tree level masses are

$$m_{h,H}^2 = \frac{1}{2}\left((m_Z^2 + m_A^2) \mp \sqrt{(m_Z^2 + m_A^2)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta}\right), \tag{35}$$

they correspond to the physical Higgs mass eigenstates

$$\begin{pmatrix} h \\ H \end{pmatrix} = \sqrt{2}\begin{pmatrix} -\sin\alpha & \cos\alpha \\ \cos\alpha & \sin\alpha \end{pmatrix}\left[\begin{pmatrix} \mathrm{Re}H_1^0 \\ \mathrm{Re}H_2^0 \end{pmatrix} - \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}\right], \tag{36}$$

$$A = \sqrt{2}\begin{pmatrix} \sin\beta \\ \cos\beta \end{pmatrix}\begin{pmatrix} \mathrm{Im}H_1^0 \\ \mathrm{Im}H_2^0 \end{pmatrix}. \tag{37}$$

The mixing angle $\alpha$ is determined by

$$\tan 2\alpha = \tan 2\beta \frac{m_A^2 + m_Z^2}{m_A^2 - m_Z^2}. \tag{38}$$

## 2.2 The neutralino sector

Although the neutralino sector does not depend on the parameters $A_\lambda$ and $A_k$ of the supersymmetry breaking potential, neutralino and Higgs sector of the NMSSM are nevertheless strongly correlated contrary to the MSSM. With fixed parameters of the Higgs sector the masses and mixings of the neutralinos are determined by the two further parameters $M$ and $M'$ of the Lagrangian

$$\begin{aligned} \mathcal{L}_{m_\chi^0} = {} & \frac{1}{\sqrt{2}} i g \lambda^3 (v_1 \psi_{H_1}^1 - v_2 \psi_{H_2}^2) - \frac{1}{\sqrt{2}} i g' \lambda' (v_1 \psi_{H_1}^1 - v_2 \psi_{H_2}^2) \\ & - \frac{1}{2} M \lambda^3 \lambda^3 - \frac{1}{2} M' \lambda' \lambda' \\ & - \lambda x \psi_{H_1}^1 \psi_{H_2}^2 - \lambda v_1 \psi_{H_2}^2 \psi_N - \lambda v_2 \psi_{H_1}^1 \psi_N + k x \psi_N^2 \\ & + \text{h.c.} \quad . \end{aligned} \tag{39}$$

In the basis

$$(\psi^0)^T = (-i\lambda_\gamma, -i\lambda_Z, \psi_H^a, \psi_H^b, \psi_N), \tag{40}$$

with

$$\begin{aligned}
\psi_H^a &= \psi_{H_1}^1 \cos\beta - \psi_{H_2}^2 \sin\beta, \\
\psi_H^b &= \psi_{H_1}^1 \sin\beta + \psi_{H_2}^2 \cos\beta,
\end{aligned} \tag{41}$$

the mass term of the Lagrangian reads

$$\mathcal{L}_{m_{\chi^0}} = -\frac{1}{2}(\psi^0)^T Y \psi^0 + \text{h.c.} \ . \tag{42}$$

The symmetric neutralino mixing matrix

$$Y = \begin{pmatrix} -Ms_W^2 - M'c_W^2 & (M'-M)s_W c_W & 0 & 0 & 0 \\ (M'-M)s_W c_W & -Mc_W^2 - M's_W^2 & m_Z & 0 & 0 \\ 0 & m_Z & -\lambda x \sin 2\beta & \lambda x \cos 2\beta & 0 \\ 0 & 0 & \lambda x \cos 2\beta & \lambda x \sin 2\beta & \lambda v \\ 0 & 0 & 0 & \lambda v & -2kx \end{pmatrix} \tag{43}$$

can be diagonalized by a unitary $5 \times 5$ matrix $N$

$$m_{\chi_i^0} \delta_{ij} = N_{im}^* Y_{mn} N_{jn} \ , \tag{44}$$

with real and positive mass eigenvalues $m_{\chi_i^0}$. If one tolerates negative eigenvalues, the diagonalization matrix can be chosen to be real. Then the absolute values of $m_{\chi_i^0} \gtrless 0$ are the physical neutralino masses. The upper $4 \times 4$ matrix of $Y$ represents the neutralino mixing matrix of the MSSM with $\mu = \lambda x$. The negative sign of the parameters $M$ and $M'$ and $\mu$ in $Y$ opposite to the convention in ref. [37] leaves all neutralino physics unchanged.

As in the MSSM, from the two-component mass eigenstates

$$\chi_i^0 = N_{ij} \psi_j^0, \qquad (i, j = 1, \ldots, 5), \tag{45}$$

the four-component Majorana spinors are formed by

$$\tilde{\chi}_i^0 = \begin{pmatrix} \chi_i^0 \\ \bar{\chi}_i^0 \end{pmatrix}, \qquad (i = 1, \ldots, 5). \tag{46}$$

The Feynman rules in Sec. 3.7 are derived using these four-component spinors.

## 2.3 The chargino sector

Although the NMSSM does not extend the chargino sector of the MSSM, we include the most important definitions in order to fix our conventions. With the basis

$$\psi^+ = \left(-i\lambda^+, \psi_{H_2}^1\right), \quad \psi^- = \left(-i\lambda^-, \psi_{H_1}^2\right), \tag{47}$$

the chargino mass term in the Lagrangian is

$$\mathcal{L}_{m_{\chi^\pm}} = -\frac{1}{2}(\psi^+, \psi^-) \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix}. \tag{48}$$

The chargino mass matrix

$$X = \left( \begin{array}{cc} -M & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & -\lambda x \end{array} \right) \tag{49}$$

can be diagonalized by two unitary matrices $U$ and $V$, so that the mass eigenvalues $m_{\chi_i^\pm}$ become

$$m_{\chi_i^\pm} \delta_{ij} = U^*_{im} X_{mn} V_{jn}. \tag{50}$$

From the two-component mass eigenstates

$$\tilde{\chi}_i^+ = V_{ij} \psi_j^+, \quad \tilde{\chi}_i^- = U_{ij} \psi_j^-, \tag{51}$$

one constructs the four-component chargino fields

$$\tilde{\chi}_i^+ = \left( \begin{array}{c} \chi_i^+ \\ \bar{\chi}_i^- \end{array} \right), \qquad i = 1, 2, \tag{52}$$

which will be used in our Feynman rules. Analytic expressions for the chargino masses and mixings are given in ref. [38]. Note that we again use a different sign convention in eq. (49), which, however, does not alter the physical results.

## 2.4 The slepton/squark sector

Likewise, the mixings between the left and right-handed sleptons and squarks remain unchanged compared to the MSSM. The mass term of the Lagrangian contained in the $D$ and $F$ terms of the scalar potential and in the soft supersymmetry breaking potential reads

$$\mathcal{L}_{m_{\tilde{u}}} = -\left(\tilde{u}_L^*, \tilde{u}_R^*\right) \left( \begin{array}{cc} a & b \\ b & c \end{array} \right) \left( \begin{array}{c} \tilde{u}_L \\ \tilde{u}_R \end{array} \right) \tag{53}$$

with

$$\begin{aligned} a &= m_Q^2 + m_u^2 + m_Z^2 \cos 2\beta \left( \frac{1}{2} - e_u \sin^2\theta_W \right), && (54) \\ b &= m_u \left( \lambda x \cot\beta + A_U \right), && (55) \\ c &= m_U^2 + m_u^2 + e_u m_Z^2 \cos 2\beta \sin^2\theta_W, && (56) \end{aligned}$$

for scalar up-type quarks (or sleptons with isospin $+1/2$), and

$$\mathcal{L}_{m_{\tilde{d}}} = -\left(\tilde{d}_L^*, \tilde{d}_R^*\right) \left( \begin{array}{cc} a' & b' \\ b' & c' \end{array} \right) \left( \begin{array}{c} \tilde{d}_L \\ \tilde{d}_R \end{array} \right) \tag{57}$$

with

$$\begin{aligned} a' &= m_Q^2 + m_d^2 - m_Z^2 \cos 2\beta \left( \frac{1}{2} + e_d \sin^2\theta_W \right), && (58) \\ b' &= m_d \left( \lambda x \tan\beta + A_D \right), && (59) \\ c' &= m_D^2 + m_d^2 + e_d m_Z^2 \cos 2\beta \sin^2\theta_W, && (60) \end{aligned}$$

for scalar down-type quarks (or sleptons with isospin $-1/2$). The mass eigenvalues are

$$m^2_{\tilde{q}_{1,2}} = \frac{1}{2}\left(a^{(\prime)} + c^{(\prime)} \mp \sqrt{(a^{(\prime)} - c^{(\prime)})^2 + 4b^{(\prime)2}}\right), \tag{61}$$

and the mass eigenstates $\tilde{q}_1$ and $\tilde{q}_2$ read

$$\begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix} = \begin{pmatrix} \cos\theta^{(\prime)} & -\sin\theta^{(\prime)} \\ \sin\theta^{(\prime)} & \cos\theta^{(\prime)} \end{pmatrix} \begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} \tag{62}$$

with

$$\sin\theta^{(\prime)} = \frac{1}{\sqrt{1 + d^{(\prime)2}}}, \quad \cos\theta^{(\prime)} = -d^{(\prime)}\sin\theta^{(\prime)}, \tag{63}$$

and

$$d^{(\prime)} = \frac{b^{(\prime)}}{a^{(\prime)} - m^2_{\tilde{q}_1}}. \tag{64}$$

# 3 Feynman rules

In this section we give a complete set of Feynman rules for all vertex factors in the NMSSM that differ from those of the MSSM. Therefore it contains all Feynman rules with scalar or pseudoscalar Higgs particles as well as the $\tilde{\chi}^0\tilde{\chi}^{\pm}C^{\mp}$ coupling. Since the charged Higgs sector of the NMSSM is not enlarged, all further vertex functions with charged Higgs bosons but without neutral Higgs particles remain unchanged in the NMSSM. The corresponding Feynman rules for the MSSM can be found e. g. in refs. [27, 28].

Differences between the vertex functions of NMSSM and MSSM can arise because of two reasons: First, the Feynman rules can be formally identical, but the Higgs mixings are different. Generally, in this case the vertex factor is suppressed in the NMSSM when the Higgs bosons have a significant singlet component. Second, the Feynman rules can differ explicitly if they contain terms with the Higgs singlet components $U^S_{a3}$, $U^P_{\alpha 3}$, or certain terms proportional to $\lambda$ or $k$. Then the vertex factor can be suppressed or enhanced depending on the choice of the parameters.

In the NMSSM as well as in the MSSM, Higgs bosons interact with gauge bosons, quarks, leptons, other Higgs bosons and their supersymmetric partners. We first give those pieces of the Lagrangian that are responsible for the coupling and then derive the Feynman rules for the respective vertex functions in the unitary gauge.

## 3.1 Interaction of two Higgs bosons with one gauge boson

The relevant part of the Lagrangian is

$$\begin{aligned}
\mathcal{L}_{HHV} \;=\; & -\frac{ig}{\sqrt{2}}[W^+_\mu(H_1^{1*}\overleftrightarrow{\partial}^\mu H_1^2 + H_2^{1*}\overleftrightarrow{\partial}^\mu H_2^2) + \text{h.c.}] \\
& -\frac{ig}{2\cos\theta_W}Z_\mu[(H_1^{1*}\overleftrightarrow{\partial}^\mu H_1^1 - H_2^{2*}\overleftrightarrow{\partial}^\mu H_2^2) + \\
& \qquad (-1 + 2\sin\theta_W)(H_1^{2*}\overleftrightarrow{\partial}^\mu H_1^2 + H_2^{2*}\overleftrightarrow{\partial}^\mu H_2^2)] \\
& +ieA_\mu(H_1^{2*}\overleftrightarrow{\partial}^\mu H_1^2 + H_2^{1*}\overleftrightarrow{\partial}^\mu H_2^1).
\end{aligned} \tag{65}$$

From eq. (65) we get the Lagrangian for the interaction between

1. one scalar and one pseudoscalar Higgs boson and one $Z$ boson

$$\mathcal{L}_{S_a P_\alpha Z} = \tfrac{g}{2\cos\theta_W} Z_\mu \quad (U^S_{a1} S_a \overleftrightarrow{\partial}^\mu U^P_{\alpha 1} P_\alpha - U^S_{a2} S_a \overleftrightarrow{\partial}^\mu U^P_{\alpha 2} P_\alpha)\ ; \tag{66}$$

2. one neutral scalar and one charged Higgs boson and one $W$ boson

$$\mathcal{L}_{S_a C W} = -\frac{ig}{2} W^+_\mu (\sin\beta\, U^S_{a1} S_a \overleftrightarrow{\partial}^\mu C^- - \cos\beta\, U^S_{a2} S_a \overleftrightarrow{\partial}^\mu C^-) + \text{h.c.}\ ; \tag{67}$$

3. one neutral pseudoscalar and one charged Higgs boson and one $W$ boson

$$\mathcal{L}_{P_\alpha C W} = -\frac{g}{2} (\sin\beta\, U^P_{\alpha 1} P_\alpha \overleftrightarrow{\partial}^\mu C^- + \cos\beta\, U^P_{\alpha 2} P_\alpha \overleftrightarrow{\partial}^\mu C^-) + \text{h.c.}\ . \tag{68}$$

As in the MSSM, Bose symmetry forbids the coupling of the $Z$ boson to two identical Higgs bosons, while CP invariance forbids $ZS_aS_b$ $(a \neq b)$ and $ZP_\alpha P_\beta$ $(\alpha \neq \beta)$ vertices. The Feynman rules for the vertices with two Higgs bosons and one gauge boson are given in Fig. 1.

## 3.2 Interaction of Higgs bosons with two gauge bosons

These interactions are contained in

$$\begin{aligned}\mathcal{L}_{H(H)VV} = {} & \frac{g^2}{2} W^{\mu+} W^-_\mu (|H^1_1|^2 + |H^2_1|^2 + |H^1_2|^2 + |H^2_2|^2) \\ & -\frac{g}{\sqrt{2}} (eA^\mu - \frac{g\sin^2\theta_W}{\cos\theta_W} Z^\mu)[W^+_\mu (H^{1*}_1 H^2_1 - H^{1*}_2 H^2_2) + \text{h.c.}] \\ & +eA_\mu A^\mu (|H^2_1|^2 + |H^1_2|^2) \\ & +\frac{g^2}{4\cos^2\theta_W} Z_\mu Z^\mu (|H^1_1|^2 + |H^2_2|^2 + \cos^2 2\theta_W (|H^2_1|^2 + |H^1_2|^2)) \\ & +\frac{eg}{\cos\theta_W} A_\mu Z^\mu \cos 2\theta_W (|H^2_1|^2 + |H^1_2|^2).\end{aligned} \tag{69}$$

Substituting the Higgs mass eigenstates of eqs. (31) and (32) one obtains for the trilinear interaction between

1. one scalar Higgs boson and two $Z$ bosons

$$\mathcal{L}_{S_a ZZ} = \frac{g m_Z}{2\cos\theta_W} Z_\mu Z^\mu (\cos\beta U^S_{a1} + \sin\beta U^S_{a2}) S_a; \tag{70}$$

2. one scalar Higgs boson and two $W$ bosons

$$\mathcal{L}_{S_a WW} = g m_W W^{+\mu} W^-_\mu (\cos\beta U^S_{a1} + \sin\beta U^S_{a2}) S_a. \tag{71}$$

The corresponding Feynman rules are shown in Fig. 2. All other trilinear $HVV$ couplings vanish at tree level. Since the singlet Higgs field does not couple to gauge bosons, the Feynman rules for the trilinear couplings of Secs. 3.1 and 3.2 differ from those of the minimal model only by the mixings of the Higgs bosons. For the case of a Higgs with a large singlet component, the coupling to gauge bosons may become so small that production of a singlet like scalar or pseudoscalar Higgs boson via the $ZS_aP_\alpha$ or $ZZS_a$ vertex is suppressed. We discuss the implications for the Higgs search in Sec. 4.

The quartic couplings, however, show significant differences between the minimal and nonminimal model. From the interaction Lagrangian eq. (69) one derives for the interaction of

1. two neutral scalar Higgs bosons and two $W$ bosons

$$\mathcal{L}_{S_aS_bWW} = \frac{g^2}{4}(2-\delta_{ab})(U^S_{a1}U^S_{b1} + U^S_{a2}U^S_{b2})W^{\mu+}W^-_\mu S_aS_b; \tag{72}$$

2. two neutral pseudoscalar Higgs bosons and two $W$ bosons

$$\mathcal{L}_{P_\alpha P_\beta WW} = \frac{g^2}{4}(2-\delta_{ab})(U^P_{\alpha1}U^P_{\beta1} + U^P_{\alpha2}U^P_{\beta2})W^{\mu+}W^-_\mu P_\alpha P_\beta; \tag{73}$$

3. two neutral scalar Higgs bosons and two $Z$ bosons

$$\mathcal{L}_{S_aS_bZZ} = \frac{g^2}{8\cos^2\theta_W}(2-\delta_{ab})(U^S_{a1}U^S_{b1} + U^S_{a2}U^S_{b2})Z^\mu Z_\mu S_aS_b; \tag{74}$$

4. two neutral pseudoscalar Higgs bosons and two $Z$ bosons

$$\mathcal{L}_{P_\alpha P_\beta ZZ} = \frac{g^2}{8\cos^2\theta_W}(2-\delta_{ab})(U^P_{\alpha1}U^P_{\beta1} + U^P_{\alpha2}U^P_{\beta2})Z^\mu Z_\mu P_\alpha P_\beta; \tag{75}$$

5. one neutral scalar Higgs boson, one charged Higgs particle, one $Z$ and one $W$ boson

$$\mathcal{L}_{S_aCZW} = +\frac{g^2}{2}\frac{\sin^2\theta_W}{\cos\theta_W}(U^S_{a1}\sin\beta - U^S_{a2}\cos\beta)Z^\mu W^+_\mu C^- S_a + \text{h.c.}; \tag{76}$$

6. one neutral pseudoscalar Higgs boson, one charged Higgs particle, one $Z$ and one $W$ boson

$$\mathcal{L}_{P_\alpha CZW} = -\frac{ig^2}{2}\frac{\sin^2\theta_W}{\cos\theta_W}(U^P_{\alpha1}\sin\beta + U^P_{\alpha2}\cos\beta)Z^\mu W^+_\mu C^- P_\alpha + \text{h.c.}; \tag{77}$$

7. one neutral scalar Higgs boson, one charged Higgs particle, one photon and one $W$ boson

$$\mathcal{L}_{S_aC\gamma W} = -\frac{eg}{2}(U^S_{a1}\sin\beta - U^S_{a2}\cos\beta)A^\mu W^+_\mu C^- S_a + \text{h.c.}; \tag{78}$$

8. one neutral pseudoscalar Higgs boson, one charged Higgs particle, one photon and one $W$ boson

$$\mathcal{L}_{P_\alpha C\gamma W} = \frac{ieg}{2}(U^P_{\alpha 1}\sin\beta + U^P_{\alpha 2}\cos\beta)A^\mu W^+_\mu C^- P_\alpha + \text{h.c.} \quad . \tag{79}$$

The Feynman rules for these vertex function can be found in Figs. 3 and 4. Contrary to the NMSSM, the vertices with two different neutral scalar Higgs bosons and two gauge bosons eqs. (72) and (74) vanish in the MSSM due to the orthogonality of the $2\times 2$ MSSM diagonalization matrix.

### 3.3 Interaction of Higgs bosons with quarks and leptons

The part of the Lagrangian containing the terms responsible for the masses of the quarks and their couplings to Higgs bosons reads in two-component notation

$$\begin{aligned} \mathcal{L}_{Hq\bar{q}} &= -h_d(H_1^1 Q^2 D - H_1^2 Q^1 D) \\ &\quad -h_u(H_2^2 Q^1 U - H_2^1 Q^2 U) + \text{h.c.} \quad . \end{aligned} \tag{80}$$

Introducing four-component spinors

$$u = \begin{pmatrix} Q^1 \\ U \end{pmatrix}, \qquad d = \begin{pmatrix} Q^2 \\ D \end{pmatrix}, \tag{81}$$

one finds besides the relation between the coupling parameters $h_{u,d}$ and the quark masses $m_{u,d}$

$$h_u = \frac{g m_u}{\sqrt{2} m_W \sin\beta}, \tag{82}$$

$$h_d = \frac{g m_d}{\sqrt{2} m_W \cos\beta}, \tag{83}$$

the trilinear interaction terms

$$\mathcal{L}_{S_a u\bar{u}} = -\frac{g m_u}{2 m_W \sin\beta} U^S_{a2} S_a \bar{u} u, \tag{84}$$

$$\mathcal{L}_{S_a d\bar{d}} = -\frac{g m_d}{2 m_W \cos\beta} U^S_{a1} S_a \bar{d} d, \tag{85}$$

$$\mathcal{L}_{P_\alpha u\bar{u}} = \frac{i g m_u}{2 m_W \sin\beta} U^P_{\alpha 2} P_\alpha \bar{u}\gamma_5 u, \tag{86}$$

$$\mathcal{L}_{P_\alpha d\bar{d}} = \frac{i g m_d}{2 m_W \cos\beta} U^P_{\alpha 1} P_\alpha \bar{d}\gamma_5 d. \tag{87}$$

As in the MSSM, up-type quarks couple to the $H_2$ component of the respective Higgs boson, while the coupling to down-type quarks contains the $H_1$ component. The relevant Feynman rules are displayed in Fig. 5. The interactions with leptons are obtained by the replacement $(u,d) \longrightarrow (\nu, e)$. The generalization to three generations proceeds in the same way as in the MSSM [27].

| | $\tilde{q}_L\tilde{q}'_L$ | $\tilde{q}_L\tilde{q}'_R$ | $\tilde{q}'_L\tilde{q}_R$ | $\tilde{q}_R\tilde{q}'_R$ |
|---|---|---|---|---|
| $\tilde{q}_1\tilde{q}'_1$ | $\cos\theta\cos\theta'$ | $\cos\theta\sin\theta'$ | $\cos\theta'\sin\theta$ | $\sin\theta\sin\theta'$ |
| $\tilde{q}_1\tilde{q}'_2$ | $-\cos\theta\sin\theta'$ | $\cos\theta\cos\theta'$ | $-\sin\theta\sin\theta'$ | $\cos\theta'\sin\theta$ |
| $\tilde{q}_2\tilde{q}'_2$ | $\sin\theta\sin\theta'$ | $-\sin\theta\cos\theta'$ | $-\sin\theta'\cos\theta'$ | $\cos\theta\cos\theta'$ |

Table 1: Coefficients that convert Feynman rules for the vertex functions with two scalar quarks from the $\tilde{q}_L - \tilde{q}_R$ basis to the $\tilde{q}_1 - \tilde{q}_2$ basis. The squark mixing angles $\theta$ are defined in Sec. 2.4.

### 3.4 Interaction of Higgs bosons with scalar quarks

These interactions arise from the following $D$ and $F$ terms of the scalar potential as well as from the supersymmetry breaking terms:

$$
\begin{aligned}
\mathcal{L}^D_{H\tilde{q}\tilde{q}} &= (H_2^{2*}H_2^2 - H_1^{1*}H_1^1)\Big\{\frac{g^2}{4}(\tilde{Q}^{1*}\tilde{Q}^1 - \tilde{Q}^{2*}\tilde{Q}^2) \\
&\quad -\frac{g'^2}{4}[(y_q(\tilde{Q}^{1*}\tilde{Q}^1 + \tilde{Q}^{2*}\tilde{Q}^2) + y_u\tilde{U}^*\tilde{U} + y_d\tilde{D}^*\tilde{D})]\Big\} \\
&\quad -\frac{g^2}{2}[H_1^1\tilde{Q}^{1*}H_1^{2*}\tilde{Q}^2 + H_2^1\tilde{Q}^{1*}H_2^{2*}\tilde{Q}^2 + \text{h.c.}], \qquad (88)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}^F_{H\tilde{q}\tilde{q}} &= -|h_u\tilde{Q}^1H_2^2|^2 - |h_u\tilde{U}H_2^2|^2 - |h_d\tilde{Q}^2H_1^1|^2 - |h_d\tilde{D}H_1^1|^2 \\
&\quad +\{-\lambda H_2^2Nh_d\tilde{Q}^{2*}\tilde{D}^* - \lambda H_1^1Nh_u\tilde{Q}^{1*}\tilde{U}^* \\
&\quad -\lambda H_2^1Nh_d\tilde{Q}^{1*}\tilde{D}^* - \lambda H_1^{2*}N^*h_u\tilde{Q}^2\tilde{U} \\
&\quad +h_uh_d(H_1^{2*}H_2^2\tilde{U}\tilde{D}^* + H_2^1H_1^{1*}\tilde{U}\tilde{D}^*) \\
&\quad +h_u^2H_2^1H_2^{2*}\tilde{Q}^{1*}\tilde{Q}^2 + h_d^2H_1^{2*}H_1^1\tilde{Q}^{1*}\tilde{Q}^2 + \text{h.c.}\}, \qquad (89)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}^{\text{soft}}_{H\tilde{q}\tilde{q}} &= -h_uA_U\tilde{Q}^1\tilde{U}H_2^2 - h_dA_D\tilde{Q}^2\tilde{D}H_1^1 \\
&\quad +(h_uA_UH_2^1\tilde{Q}^2\tilde{U} + h_dA_DH_1^{2*}H_1^1\tilde{Q}^{1*}\tilde{D}^* + \text{h.c.})\ . \qquad (90)
\end{aligned}
$$

We list in the following the trilinear and quartic interactions of Higgs bosons with the weak interaction eigenstates $\tilde{q}_L$, $\tilde{q}_R$. The mass eigenstates of the squarks and sleptons are obtained with the transformation eq. (62), so that the vertex functions can be converted with

$$V(\tilde{q}_i\tilde{q}'_j) = \sum_{k,l=L,R} C_{ijkl}V(\tilde{q}_k\tilde{q}'_l). \qquad (91)$$

In eq. (91), $V(\tilde{q}_i\tilde{q}'_j)$ denotes any vertex function with two scalar quarks $\tilde{q}_i$ and $\tilde{q}'_j$ $(i,j = 1,2)$. The coefficients $C_{ijkl}$ are given in Table 1.

In the $q_L - q_R$ basis we derive from eqs. (88) – (90) the trilinear interactions for

1. one scalar Higgs boson and two left-handed up-type squarks

$$
\begin{aligned}
\mathcal{L}_{S_a\tilde{u}_L\tilde{u}_L} &= [-\frac{gm_u^2}{m_W\sin\beta}U^S_{a2} \\
&\quad +\frac{g}{2}\frac{m_Z}{\cos\theta_W}(1-2e_u\sin^2\theta_W)(\sin\beta U^S_{a2} - \cos\beta U^S_{a1})]S_a\tilde{u}^*_L\tilde{u}_L; \qquad (92)
\end{aligned}
$$

2. one scalar Higgs boson and two left-handed down-type squarks

$$\mathcal{L}_{S_a\tilde{d}_L\tilde{d}_L} = [-\frac{gm_d^2}{m_W\cos\beta}U^S_{a1} \\ -\frac{g}{2}\frac{m_Z}{\cos\theta_W}(1+2e_d\sin^2\theta_W)(\sin\beta U^S_{a2}-\cos\beta U^S_{a1})]S_a\tilde{d}^*_L\tilde{d}_L; \quad (93)$$

3. one scalar Higgs boson and two right-handed up-type squarks

$$\mathcal{L}_{S_a\tilde{u}_R\tilde{u}_R} = [-\frac{gm_u^2}{m_W\sin\beta}U^S_{a2} \\ +gm_We_u\tan^2\theta_W(\sin\beta U^S_{a2}-\cos\beta U^S_{a1})]S_a\tilde{u}^*_R\tilde{u}_R; \quad (94)$$

4. one scalar Higgs boson and two right-handed down-type squarks

$$\mathcal{L}_{S_a\tilde{d}_R\tilde{d}_R} = [-\frac{gm_d^2}{m_W\cos\beta}U^S_{a1} \\ +gm_We_d\tan^2\theta_W(\sin\beta U^S_{a2}-\cos\beta U^S_{a1})]S_a\tilde{d}^*_R\tilde{d}_R; \quad (95)$$

5. one scalar Higgs boson and one left-handed and one right-handed up-type squark

$$\mathcal{L}_{S_a\tilde{u}_L\tilde{u}_R} = -\frac{gm_u}{2m_W\sin\beta}\left(\lambda\left(v_1U^S_{a3}+xU^S_{a1}\right)+A_UU^S_{a2}\right)S_a\tilde{u}^*_R\tilde{u}_L+\text{h.c.}\ ; \quad (96)$$

6. one scalar Higgs boson and one left-handed and one right-handed down-type squark

$$\mathcal{L}_{S_a\tilde{d}_L\tilde{d}_R} = -\frac{gm_d}{2m_W\cos\beta}\left(\lambda\left(v_2U^S_{a3}+xU^S_{a2}\right)+A_DU^S_{a1}\right)S_a\tilde{d}^*_R\tilde{d}_L+\text{h.c.}\ ; \quad (97)$$

7. one pseudoscalar Higgs boson and one left-handed and one right-handed up-type squark

$$\mathcal{L}_{P_\alpha\tilde{u}_L\tilde{u}_R} = \frac{igm_u}{2m_W\sin\beta}\left(\lambda\left(v_1U^P_{\alpha3}+xU^P_{\alpha1}\right)-A_UU^P_{\alpha2}\right)P_\alpha\tilde{u}^*_R\tilde{u}_L+\text{h.c.}\ ; \quad (98)$$

8. one pseudoscalar Higgs boson and one left-handed and one right-handed down-type squark

$$\mathcal{L}_{P_\alpha\tilde{d}_L\tilde{d}_R} = \frac{igm_d}{2m_W\cos\beta}\left(\lambda\left(v_2U^P_{\alpha3}+xU^P_{\alpha2}\right)-A_DU^P_{\alpha1}\right)P_\alpha\tilde{d}^*_R\tilde{d}_L+\text{h.c.}\ . \quad (99)$$

The corresponding Feynman rules are shown in Figs. 6 – 8. The vertex factors with one right-handed and one left-handed scalar quark in eqs. (96) – (99) explicitly depend on the singlet components $U^S_{a3}$, $U^P_{\alpha3}$ of the neutral Higgs bosons. Therefore these couplings could be enhanced in the NMSSM compared to the minimal model.

In the same way one obtains for the quartic interactions of

1. two scalar Higgs bosons and two left-handed up-type squarks

$$\mathcal{L}_{S_a S_b \tilde{u}_L \tilde{u}_L} = \frac{g^2}{8}\Bigg[\left(\frac{1}{\cos^2\theta_W} - 2e_u \tan^2\theta_W\right)(U^S_{a2}U^S_{b2} - U^S_{a1}U^S_{b1}) \\ -2\frac{m_u^2}{m_W^2 \sin^2\beta}U^S_{a2}U^S_{b2}\Bigg](2-\delta_{ab})S_a S_b \tilde{u}_L^* \tilde{u}_L; \tag{100}$$

2. two scalar Higgs bosons and two left-handed down-type squarks

$$\mathcal{L}_{S_a S_b \tilde{d}_L \tilde{d}_L} = -\frac{g^2}{8}\Bigg[\left(\frac{1}{\cos^2\theta_W} + 2e_d \tan^2\theta_W\right)(U^S_{a2}U^S_{b2} - U^S_{a1}U^S_{b1}) \\ +2\frac{m_d^2}{m_W^2 \cos^2\beta}U^S_{a1}U^S_{b1}\Bigg](2-\delta_{ab})S_a S_b \tilde{d}_L^* \tilde{d}_L; \tag{101}$$

3. two scalar Higgs bosons and two right-handed up-type squarks

$$\mathcal{L}_{S_a S_b \tilde{u}_R \tilde{u}_R} = \frac{g^2}{4}\Bigg[e_u \tan^2\theta_W (U^S_{a2}U^S_{b2} - U^S_{a1}U^S_{b1}) \\ -\frac{m_u^2}{m_W^2 \sin^2\beta}U^S_{a2}U^S_{b2}\Bigg](2-\delta_{ab})S_a S_b \tilde{u}_R^* \tilde{u}_R; \tag{102}$$

4. two scalar Higgs bosons and two right-handed down-type squarks

$$\mathcal{L}_{S_a S_b \tilde{d}_R \tilde{d}_R} = \frac{g^2}{4}\Bigg[e_d \tan^2\theta_W (U^S_{a2}U^S_{b2} - U^S_{a1}U^S_{b1}) \\ -\frac{m_d^2}{m_W^2 \cos^2\beta}U^S_{a1}U^S_{b1}\Bigg](2-\delta_{ab})S_a S_b \tilde{d}_R^* \tilde{d}_R; \tag{103}$$

5. two scalar Higgs bosons and one left-handed and one right-handed up-type squark

$$\mathcal{L}_{S_a S_b \tilde{u}_L \tilde{u}_R} = -\lambda\frac{g m_u}{4\sqrt{2} m_W \sin\beta}(U^S_{a1}U^S_{b3} + U^S_{b1}U^S_{a3})(2-\delta_{ab})S_a S_b \tilde{u}_L^* \tilde{u}_R \\ +\text{h.c.}\ ; \tag{104}$$

6. two scalar Higgs bosons and one left-handed and one right-handed down-type squark

$$\mathcal{L}_{S_a S_b \tilde{d}_L \tilde{d}_R} = -\lambda\frac{g m_d}{4\sqrt{2} m_W \cos\beta}(U^S_{a2}U^S_{b3} + U^S_{a3}U^S_{b2})(2-\delta_{ab})S_a S_b \tilde{d}_L^* \tilde{d}_R \\ +\text{h.c.}\ ; \tag{105}$$

7. two pseudoscalar Higgs bosons and two left-handed up-type squarks

$$\mathcal{L}_{P_\alpha P_\beta \tilde{u}_L \tilde{u}_L} = \frac{g^2}{8}\Bigg[\left(\frac{1}{\cos^2\theta_W} - 2e_u \tan^2\theta_W\right)(U^P_{\alpha 2}U^P_{\beta 2} - U^P_{\alpha 1}U^P_{\beta 1}) \\ -2\frac{m_u^2}{m_W^2 \sin^2\beta}U^P_{\alpha 2}U^P_{\beta 2}\Bigg](2-\delta_{\alpha\beta})P_\alpha P_\beta \tilde{u}_L^* \tilde{u}_L; \tag{106}$$

8. two pseudoscalar Higgs bosons and two left-handed down-type squarks

$$\mathcal{L}_{P_\alpha P_\beta \tilde{d}_L \tilde{d}_L} = \frac{g^2}{8}\Bigg[\left(\frac{1}{\cos^2\theta_W} + 2e_d\tan^2\theta_W\right)(U^P_{\alpha 2}U^P_{\beta 2} - U^P_{\alpha 1}U^P_{\beta 1}) - 2\frac{m_u^2}{m_W^2\cos^2\beta}U^P_{\alpha 1}U^P_{\beta 1}\Bigg](2-\delta_{\alpha\beta})P_\alpha P_\beta \tilde{d}^*_L\tilde{d}_L; \quad (107)$$

9. two pseudoscalar Higgs bosons and two right-handed up-type squarks

$$\mathcal{L}_{P_\alpha P_\beta \tilde{u}_R \tilde{u}_R} = \frac{g^2}{4}\Big[e_u\tan^2\theta_W(U^P_{\alpha 2}U^P_{\beta 2} - U^P_{\alpha 1}U^P_{\beta 1}) - \frac{m_u^2}{m_W^2\sin^2\beta}U^P_{\alpha 2}U^P_{\beta 2}\Big](2-\delta_{\alpha\beta})P_\alpha P_\beta \tilde{u}^*_R\tilde{u}_R; \quad (108)$$

10. two pseudoscalar Higgs bosons and two right-handed down-type squarks

$$\mathcal{L}_{P_\alpha P_\beta \tilde{d}_R \tilde{d}_R} = \frac{g^2}{4}\Big[e_d\tan^2\theta_W(U^P_{\alpha 2}U^P_{\beta 2} - U^P_{\alpha 1}U^P_{\beta 1}) - \frac{m_d^2}{m_W^2\cos^2\beta}U^P_{\alpha 1}U^P_{\beta 1}\Big](2-\delta_{\alpha\beta})P_\alpha P_\beta \tilde{d}^*_R\tilde{d}_R; \quad (109)$$

11. two pseudoscalar Higgs bosons and one left-handed and one right-handed up-type squark

$$\mathcal{L}_{P_\alpha P_\beta \tilde{u}_L \tilde{u}_R} = \lambda\frac{gm_u}{4\sqrt{2}m_W\sin\beta}(U^P_{\alpha 1}U^P_{\beta 3} + U^P_{\alpha 3}U^P_{\beta 1})(2-\delta_{\alpha\beta})P_\alpha P_\beta \tilde{u}^*_L\tilde{u}_R + \text{h.c.}\,; \quad (110)$$

12. two pseudoscalar Higgs bosons and one left-handed and one right-handed down-type squark

$$\mathcal{L}_{P_\alpha P_\beta \tilde{d}_L \tilde{d}_R} = \lambda\frac{gm_d}{4\sqrt{2}m_W\cos\beta}(U^P_{\alpha 2}U^P_{\beta 3} + U^S_{\alpha 3}U^S_{\beta 2})(2-\delta_{\alpha\beta})P_\alpha P_\beta \tilde{d}^*_L\tilde{d}_R + \text{h.c.}\,; \quad (111)$$

13. one scalar and one pseudoscalar Higgs boson and one left-handed and one right-handed up-type squark

$$\mathcal{L}_{S_a P_\alpha \tilde{u}_L \tilde{u}_R} = -i\lambda\frac{gm_u}{2\sqrt{2}m_W\sin\beta}(U^S_{a1}U^P_{\alpha 3} + U^S_{a3}U^P_{\alpha 1})S_a P_\alpha \tilde{u}^*_L\tilde{u}_R + \text{h.c.}\,; \quad (112)$$

14. one scalar and one pseudoscalar Higgs boson and one left-handed and one right-handed down-type squark

$$\mathcal{L}_{S_a P_\alpha \tilde{d}_L \tilde{d}_R} = -i\lambda\frac{gm_d}{2\sqrt{2}m_W\cos\beta}(U^S_{a2}U^P_{\alpha 3} + U^S_{a3}U^P_{\alpha 2})S_a P_\alpha \tilde{d}^*_L\tilde{d}_R + \text{h.c.}\,; \quad (113)$$

15. one neutral scalar Higgs boson, one charged Higgs boson, one left-handed up-type squark and one left-handed down-type squark

$$\begin{aligned}\mathcal{L}_{S_a C \tilde{u}_L \tilde{d}_L} &= -\frac{g^2}{2\sqrt{2}}\Big(U^S_{a1}\sin\beta + U^S_{a2}\cos\beta - \frac{m_u^2}{m_W^2}\frac{\cos\beta}{\sin^2\beta}U^S_{a2} \\ &\qquad -\frac{m_d^2}{m_W^2}\frac{\sin\beta}{\cos^2\beta}U^S_{a1}\Big)S_a C^+ \tilde{u}_L^* \tilde{d}_L + \text{h.c.}\ ; \end{aligned} \tag{114}$$

16. one neutral scalar Higgs boson, one charged Higgs boson, one right-handed up-type squark and one right-handed down-type squark

$$\mathcal{L}_{S_a C \tilde{u}_R \tilde{d}_R} = \frac{g^2 m_u m_d}{\sqrt{2} m_W^2 \sin 2\beta}(U^S_{a2}\sin\beta + U^S_{a1}\cos\beta) S_a C^+ \tilde{u}_R^* \tilde{d}_R + \text{h.c.}\ ; \tag{115}$$

17. one neutral scalar Higgs boson, one charged Higgs boson, one left-handed up-type squark and one right-handed down-type squark

$$\mathcal{L}_{S_a C \tilde{u}_L \tilde{d}_R} = -\lambda \frac{g m_d}{2 m_W} U^S_{a3} S_a C^+ \tilde{u}_L^* \tilde{d}_R + \text{h.c.}\ ; \tag{116}$$

18. one neutral scalar Higgs boson, one charged Higgs boson, one left-handed down-type squark and one right-handed up-type squark

$$\mathcal{L}_{S_a C \tilde{u}_R \tilde{d}_L} = -\lambda \frac{g m_u}{2 m_W} U^S_{a3} S_a C^+ \tilde{u}_R^* \tilde{d}_L + \text{h.c.}\ ; \tag{117}$$

19. one neutral pseudoscalar Higgs boson, one charged Higgs boson, one left-handed up-type squark and one left-handed down-type squark

$$\begin{aligned}\mathcal{L}_{P_\alpha C \tilde{u}_L \tilde{d}_L} &= -\frac{i g^2}{2\sqrt{2}}\Big(U^P_{\alpha 1}\sin\beta - U^P_{\alpha 2}\cos\beta + \frac{m_u^2}{m_W^2}\frac{\cos\beta}{\sin^2\beta}U^P_{\alpha 2} \\ &\qquad -\frac{m_d^2}{m_W^2}\frac{\sin\beta}{\cos^2\beta}U^P_{\alpha 1}\Big)P_\alpha C^+ \tilde{u}_L^* \tilde{d}_L + \text{h.c.}\ ; \end{aligned} \tag{118}$$

20. one neutral pseudoscalar Higgs boson, one charged Higgs boson, one right-handed up-type squark and one right-handed down-type squark

$$\mathcal{L}_{P_\alpha C \tilde{u}_R \tilde{d}_R} = i\frac{g^2 m_u m_d}{\sqrt{2} m_W^2 \sin 2\beta}(U^P_{\alpha 2}\sin\beta - U^P_{\alpha 1}\cos\beta) P_\alpha C^+ \tilde{u}_R^* \tilde{d}_R + \text{h.c.}\ ; \tag{119}$$

21. one neutral pseudoscalar Higgs boson, one charged Higgs boson, one left-handed up-type squark and one right-handed down-type squark

$$\mathcal{L}_{P_\alpha C \tilde{u}_L \tilde{d}_R} = -i\lambda \frac{g m_d}{2 m_W} U^P_{\alpha 3} P_\alpha C^+ \tilde{u}_L^* \tilde{d}_R + \text{h.c.}\ ; \tag{120}$$

22. one neutral pseudoscalar Higgs boson, one charged Higgs boson, one left-handed down-type squark and one right-handed up-type squark

$$\mathcal{L}_{P_\alpha C \tilde{u}_R \tilde{d}_L} = i\lambda \frac{g m_u}{2m_W} U^P_{\alpha 3} P_\alpha C^+ \tilde{u}^*_R \tilde{d}_L + \text{h.c.} \quad . \tag{121}$$

The Feynman rules for these vertices with two Higgs bosons and two scalar quarks are given in Figs. 9 – 15. Similar as for the trilinear vertex functions, the couplings of two neutral scalar or pseudoscalar Higgs bosons to one left-handed and one right-handed scalar quark in eqs. (104), (105), (110) – (113) are explicitly affected by the singlet components of the respective Higgs particles. Moreover, one left-handed and one right-handed squark together with a charged Higgs boson couple only to the singlet component $U^S_{a3}$, $U^P_{\alpha 3}$ of a neutral Higgs boson in eqs. (116), (117), (120), (121). Since these couplings vanish in the MSSM, their existence could represent a unique test of the NMSSM.

## 3.5 Trilinear self-interaction of Higgs bosons

The self-interactions of the Higgs bosons are generated by the following parts of the $D$ and $F$ terms and the supersymmetry breaking terms of the scalar potential:

$$\mathcal{L}^D_{HHH(H)} = -\frac{1}{8}(g^2+g'^2)(H^{1*}_1 H^1_1 H^{1*}_1 H^1_1 + H^{2*}_2 H^2_2 H^{2*}_2 H^2_2 - 2H^{1*}_1 H^1_1 H^{2*}_2 H^2_2), \tag{122}$$

$$\begin{aligned}\mathcal{L}^F_{HHH(H)} &= -\lambda^2 H^{1*}_1 H^1_1 H^{2*}_2 H^2_2 - k^2 N^* N N^* N \\ &\quad +\lambda k (H^1_1 H^2_2 N^* N^* + \text{h.c.}) \\ &\quad -\lambda^2 N N^* (H^{1*}_1 H^1_1 + H^{2*}_2 H^2_2),\end{aligned} \tag{123}$$

$$\mathcal{L}^{\text{soft}}_{HHH} = \lambda A_\lambda H^1_1 H^2_2 N + \frac{1}{3} k A_k N^3 + \text{h.c.} \quad . \tag{124}$$

Inserting the mass eigenstates of eqs. (31) – (33) we find for the self-coupling of three scalar Higgs bosons

$$\begin{aligned}\mathcal{L}_{SSS} = \Big[ & -\frac{1}{4}\frac{g^2+g'^2}{\sqrt{2}}(v_1 U^S_{a1}U^S_{b1}U^S_{c1} + v_2 U^S_{a2}U^S_{b2}U^S_{c2}) \\ & + \left(\frac{g^2+g'^2}{4\sqrt{2}} - \frac{\lambda^2}{\sqrt{2}}\right)(v_1 U^S_{a1}U^S_{b2}U^S_{c2} + v_2 U^S_{a1}U^S_{b1}U^S_{c2}) \\ & + \frac{1}{\sqrt{2}}(\lambda k v_2 - \lambda^2 v_1) U^S_{a1}U^S_{b3}U^S_{c3} \\ & + \frac{1}{\sqrt{2}}(\lambda k v_1 - \lambda^2 v_2) U^S_{a2}U^S_{b3}U^S_{c3} \\ & - \frac{1}{\sqrt{2}}\lambda^2 x (U^S_{a1}U^S_{b1}U^S_{c3} + U^S_{a2}U^S_{b2}U^S_{c3}) \\ & + \lambda\left(\frac{A_\lambda}{\sqrt{2}} + \sqrt{2}kx\right) U^S_{a1}U^S_{b2}U^S_{c3} \\ & + (\frac{1}{3\sqrt{2}}k A_k - \sqrt{2}k^2) U^S_{a3}U^S_{b3}U^S_{c3}\Big] S_a S_b S_c \quad .\end{aligned} \tag{125}$$

In eq. (125) summation over the indices $a$, $b$, and $c$ still has to be carried out, which is already performed for the coupling of one scalar with two pseudoscalar Higgs bosons

$$\begin{aligned}
\mathcal{L}_{S_a P_\beta P_\gamma} = \Big[ & -\frac{g^2+g'^2}{4\sqrt{2}}\left(v_1 U^S_{a1} U^P_{\beta 1} U^P_{\gamma 1} + v_2 U^S_{a2} U^P_{\beta 2} U^P_{\gamma 2}\right) \\
& + \left(\frac{g^2+g'^2}{4\sqrt{2}} - \frac{\lambda^2}{\sqrt{2}}\right)\left(v_1 U^S_{a1} U^P_{\beta 2} U^P_{\gamma 2} + v_2 U^S_{a2} U^P_{\beta 1} U^P_{\gamma 1}\right) \\
& - \frac{1}{\sqrt{2}}(\lambda k v_1 + \lambda^2 v_2) U^S_{a2} U^P_{\beta 3} U^P_{\gamma 3} \\
& - \frac{1}{\sqrt{2}}(\lambda k v_2 + \lambda^2 v_1) U^S_{a1} U^P_{\beta 3} U^P_{\gamma 3} \\
& - \frac{\lambda^2 x}{\sqrt{2}} U^S_{a3}(U^P_{\beta 1} U^P_{\gamma 1} + U^P_{\beta 2} U^P_{\gamma 2}) \\
& - \left(\sqrt{2} k^2 x + \frac{k A_k}{\sqrt{2}}\right) U^S_{a3} U^P_{\beta 3} U^P_{\gamma 3} \\
& + \frac{\lambda k}{\sqrt{2}}\left(v_1 U^S_{a3}(U^P_{\beta 2} U^P_{\gamma 3} + U^P_{\beta 3} U^P_{\gamma 2}) + v_2 U^S_{a3}(U^P_{\beta 1} U^P_{\gamma 3} + U^P_{\beta 3} U^P_{\gamma 1})\right) \\
& + \left(\frac{\lambda k x}{\sqrt{2}} - \frac{\lambda A_\lambda}{2\sqrt{2}}\right)\left(U^S_{a1}(U^P_{\beta 2} U^P_{\gamma 3} + U^P_{\beta 3} U^P_{\gamma 2}) + U^S_{a2}(U^P_{\beta 1} U^P_{\gamma 3} + U^P_{\beta 3} U^P_{\gamma 1}\right) \\
& - \left(\frac{\lambda k x}{\sqrt{2}} + \frac{\lambda A_\lambda}{2\sqrt{2}}\right) U^S_{a3}(U^P_{\beta 1} U^P_{\gamma 2} + U^P_{\beta 2} U^P_{\gamma 1})\Big](2 - \delta_{\beta\gamma}) S_a P_\beta P_\gamma \quad . \qquad (126)
\end{aligned}$$

CP invariance forbids vertices with an odd number of pseudoscalar Higgs bosons.

The interaction of one neutral Higgs boson with two charged Higgs particles can be derived from the following parts of the Lagrangian

$$\begin{aligned}
\mathcal{L}^D_{H(H)CC} &= -\frac{g^2}{4}\Big[(H^2_2 H^{2*}_2 + H^1_1 H^{1*}_1)(H^1_2 H^{1*}_2 + H^2_1 H^{2*}_1) \\
&\qquad + 2(H^1_1 H^{1*}_2 H^{2*}_1 H^2_2 + \text{h.c.})\Big] \\
&\quad - \frac{g'^2}{4}(H^2_2 H^{2*}_2 - H^1_1 H^{1*}_1)(H^1_2 H^{1*}_2 - H^2_1 H^{2*}_1), \qquad (127) \\
\mathcal{L}^F_{H(H)CC} &= -\lambda k (H^2_1 H^1_2 N^* N^* + \text{h.c.}) - \lambda^2 (H^1_2 H^{1*}_2 + H^2_1 H^{2*}_1) N N^* \\
&\quad + \lambda^2 (H^1_1 H^2_2 H^{2*}_1 H^{1*}_2 + \text{h.c.}), \qquad (128) \\
\mathcal{L}^{\text{soft}}_{HCC} &= -\lambda A_\lambda (H^2_1 H^1_2 N + \text{h.c.}). \qquad (129)
\end{aligned}$$

With eqs. (31) and (33) the coupling of a scalar Higgs boson with two charged Higgs particles reads

$$\begin{aligned}
\mathcal{L}_{S_a C^+ C^-} = \Big\{ & - g m_W (U^S_{a1}\cos\beta + U^S_{a2}\sin\beta) \\
& - \frac{g m_Z}{2\cos\theta_W}\left(U^S_{a2}\sin\beta - U^S_{a1}\cos\beta\right)\cos 2\beta \\
& + \frac{\lambda^2}{\sqrt{2}}\left(v_1 U^S_{a2} + v_2 U^S_{a1}\right)\sin 2\beta
\end{aligned}$$

$$-\frac{1}{\sqrt{2}}\lambda U^S_{a3}\left[(2kx+A_\lambda)\sin 2\beta + 2\lambda x\right]\Big\}S_a C^+ C^- \quad . \tag{130}$$

The couplings of the pseudoscalar Higgs bosons with two charged Higgs bosons vanish due to CP conservation.

The Feynman rules for trilinear Higgs self-couplings of the NMSSM shown in Fig. 16 exhibit significant differences to their counterparts in the minimal model. They contain the singlet components $U^S_{a3}$, $U^P_{\alpha 3}$ as well as terms proportional to the couplings in the superpotential $\lambda$ and $k$ and to the parameters in the supersymmetry breaking potential $A_\lambda$ and $A_k$. Therefore, probing the Higgs self-coupling at high energy colliders probably represents a highly decisive test in oder to distinguish between NMSSM and MSSM.

## 3.6 Quartic self-interaction of Higgs bosons

The interactions between four neutral Higgs bosons are also contained in eqs. (122) and (123). Since again vertices with an odd number of pseudoscalar Higgs bosons are forbidden by CP invariance, one obtains for the interactions of

1. four scalar neutral Higgs bosons

$$\begin{aligned}\mathcal{L}_{SSSS} = \Big[ &-\frac{1}{32}(g^2+g'^2)(U^S_{a1}U^S_{b1}U^S_{c1}U^S_{d1}+U^S_{a2}U^S_{b2}U^S_{c2}U^S_{d2}) \\ &+\left(\frac{1}{16}(g^2+g'^2)-\frac{\lambda^2}{4}\right)U^S_{a1}U^S_{b1}U^S_{c2}U^S_{d2} \\ &-\frac{\lambda^2}{4}(U^S_{a1}U^S_{b1}U^S_{c3}U^S_{d3}+U^S_{a2}U^S_{b2}U^S_{c3}U^S_{d3}) \\ &-\frac{k^2}{4}U^S_{a3}U^S_{b3}U^S_{c3}U^S_{d3}-\frac{\lambda k}{2}U^S_{a1}U^S_{b2}U^S_{c3}U^S_{d3}\Big]S_aS_bS_cS_d \; ;\end{aligned} \tag{131}$$

2. two scalar and two pseudoscalar Higgs bosons

$$\begin{aligned}\mathcal{L}_{SSPP} = \Big[ &-\frac{1}{16}(g^2+g'^2)(U^S_{a1}U^S_{b1}U^P_{\gamma 1}U^P_{\delta 1}+U^S_{a2}U^S_{b2}U^P_{\gamma 2}U^P_{\delta 2}) \\ &+\left(\frac{1}{16}(g^2+g'^2)-\frac{\lambda^2}{4}\right)(U^S_{a1}U^S_{b1}U^P_{\gamma 2}U^P_{\gamma 2}+U^S_{a2}U^S_{b2}U^P_{\gamma 1}U^P_{\delta 1}) \\ &-\frac{\lambda^2}{4}(U^S_{a1}U^S_{b1}U^P_{\gamma 3}U^P_{\delta 3}+U^S_{a3}U^S_{b3}U^P_{\gamma 1}U^P_{\delta 1}+U^S_{a2}U^S_{b2}U^P_{\gamma 3}U^P_{\delta 3}+U^S_{a3}U^S_{b3}U^P_{\gamma 2}U^P_{\delta 2}) \\ &-\frac{\lambda k}{2}(U^S_{a1}U^S_{b2}U^P_{\gamma 3}U^P_{\delta 3}+U^S_{a3}U^S_{b3}U^P_{\gamma 1}U^P_{\delta 2}-2U^S_{a1}U^S_{b3}U^P_{\gamma 2}U^P_{\delta 3}-2U^S_{a2}U^S_{b3}U^P_{\gamma 1}U^P_{\delta 3}) \\ &-\frac{k^2}{2}U^S_{a3}U^S_{b3}U^P_{\gamma 3}U^P_{\delta 3}\Big]S_aS_bP_\gamma P_\delta \; ;\end{aligned} \tag{132}$$

3. four pseudoscalar Higgs bosons

$$\mathcal{L}_{PPPP} = \Big[ -\frac{1}{32}(g^2+g'^2)(U^P_{\alpha 1}U^P_{\beta 1}U^P_{\gamma 1}U^P_{\delta 1}+U^P_{\alpha 2}U^P_{\beta 2}U^P_{\gamma 2}U^P_{\delta 2})$$

$$
\begin{aligned}
&+\left(\frac{1}{16}(g^2+g'^2)-\frac{\lambda^2}{4}\right)U^P_{\alpha1}U^P_{\beta1}U^P_{\gamma2}U^P_{\delta2}\\
&-\frac{\lambda^2}{4}(U^P_{\alpha1}U^P_{\beta1}U^P_{\gamma3}U^P_{\delta3}+U^P_{\alpha2}U^P_{\beta2}U^P_{\gamma3}U^P_{\delta3})\\
&-\frac{k^2}{4}U^P_{\alpha3}U^P_{\beta3}U^P_{\gamma3}U^P_{\delta3}-\frac{\lambda k}{2}U^P_{\alpha1}U^P_{\beta2}U^P_{\gamma3}U^P_{\delta3}\Big]P_\alpha P_\beta P_\gamma P_\delta. \qquad (133)
\end{aligned}
$$

In eqs. (131) – (133) the sum over repeated indices has to be performed. Latin indices for the scalar Higgs bosons run from 1 to 3, the greek indices of pseudoscalar Higgs bosons are summed from 1 to 2.

The interactions with two neutral and two charged Higgs particles follow directly from eqs. (127) and (128). They read for the case of scalar Higgs bosons

$$
\begin{aligned}
\mathcal{L}_{S_aS_bCC} &= \Big[-\frac{1}{8}g^2(U^S_{a2}U^S_{b2}+U^S_{a1}U^S_{b1})\\
&\quad-\left(\frac{1}{8}g^2-\frac{\lambda^2}{4}\right)(U^S_{a1}U^S_{b2}+U^S_{a2}U^S_{b1})\sin 2\beta\\
&\quad-\frac{1}{8}g'^2(U^S_{a2}U^S_{b2}-U^S_{a1}U^S_{b1})\cos 2\beta\\
&\quad-\frac{\lambda}{2}(\lambda U^S_{a3}U^S_{b3}+kU^S_{a3}U^S_{b3}\sin 2\beta)\Big](2-\delta_{ab})S_aS_bC^+C^-, \qquad (134)
\end{aligned}
$$

and for pseudoscalar Higgs bosons

$$
\begin{aligned}
\mathcal{L}_{P_\alpha P_\beta CC} &= \Big[-\frac{1}{8}g^2(U^P_{\alpha2}U^P_{\beta2}+U^P_{\alpha1}U^P_{\beta1})\\
&\quad+\left(\frac{1}{8}g^2-\frac{\lambda^2}{4}\right)(U^P_{\alpha1}U^P_{\beta2}+U^P_{\alpha2}U^P_{\beta1})\sin 2\beta\\
&\quad-\frac{1}{8}g'^2(U^P_{\alpha2}U^P_{\beta2}-U^P_{\alpha1}U^P_{\beta1})\cos 2\beta\\
&\quad-\frac{\lambda}{2}(\lambda U^P_{\alpha3}U^P_{\beta3}-kU^P_{\alpha3}U^P_{\beta3}\sin 2\beta)\Big](2-\delta_{\alpha\beta})P_\alpha P_\beta C^+C^-. \qquad (135)
\end{aligned}
$$

The Feynman rules for the quartic Higgs vertices are displayed in Fig. 17. As the trilinear Higgs interactions, also the quartic Higgs self-couplings of the NMSSM differ significantly from those of the MSSM. They are, however, of lesser importance for the supersymmetric processes tested at the particle colliders in the nearer future.

## 3.7 Interaction of Higgs bosons with neutralinos and charginos

For these interactions one has to take into account the mass eigenstates of the Higgs bosons as well as the neutralino/chargino mixing as described in Sec. 2. In four-component notation the Lagrangian for the interaction of a neutral Higgs boson with two charginos reads

$$
\mathcal{L}^{\rm int}_{H\tilde{\chi}^+\tilde{\chi}^-} = -g(H_1^{1*}\bar{\tilde{H}}P_L\tilde{W}+H_2^{2*}\bar{\tilde{W}}P_L\tilde{H})+\lambda N\bar{\tilde{H}}P_L\tilde{H}+\text{h.c.} \;. \qquad (136)
$$

Substituting the mass eigenstates of the Higgs bosons (eqs. (31) and (32)) and of the charginos (eq. (52)) one finds

$$\begin{aligned}\mathcal{L}_{H\tilde{\chi}^+\tilde{\chi}^-} &= -S_a\bar{\tilde{\chi}}_i^+\left[Q^*_{aij}P_L + Q_{aji}P_R\right]\tilde{\chi}_j^+ \\ &\quad -iP_\alpha\bar{\tilde{\chi}}_i^+\left[R^*_{\alpha ij}P_L - R_{\alpha ji}P_R\right]\tilde{\chi}_j^+ \quad , \end{aligned} \tag{137}$$

where

$$Q_{aij} = \frac{g}{\sqrt{2}}(U^S_{a1}U_{i2}V_{j1} + U^S_{a2}U_{i1}V_{j2}) - \frac{\lambda}{\sqrt{2}}U^S_{a3}U_{i2}V_{j2} \quad , \tag{138}$$

$$R_{\alpha ij} = -\frac{g}{\sqrt{2}}(U^P_{\alpha 1}U_{i2}V_{j1} + U^P_{\alpha 2}U_{i1}V_{j2}) - \frac{\lambda}{\sqrt{2}}U^P_{\alpha 3}U_{i2}V_{j2} \quad . \tag{139}$$

For real matrices $Q$ and $R$ the couplings are indeed scalar for the $S_a$ and pseudoscalar for the $P_\alpha$.

The source for the coupling of a charged Higgs boson to a neutralino and a chargino is the Lagrangian

$$\begin{aligned}\mathcal{L}^{\text{int}}_{C^\pm\tilde{\chi}^0\tilde{\chi}^\pm} &= -\frac{g}{\sqrt{2}}\left(H_2^{1*}\left(2s_W\bar{\tilde{\gamma}} + \frac{1-2s_W^2}{c_W}\bar{\tilde{Z}}\right)P_L\tilde{H} - H_1^{2*}\bar{\tilde{H}}P_L\left(2s_W\tilde{\gamma} + \frac{1-2s_W^2}{c_W}\tilde{Z}\right)\right) \\ &\quad -g\left(H_1^{2*}\bar{\tilde{W}}P_L\left(\tilde{H}_a\cos\beta + \tilde{H}_b\sin\beta\right) + H_2^{1*}\left(-\bar{\tilde{H}}_a\sin\beta + \bar{\tilde{H}}_b\cos\beta\right)P_L\tilde{W}\right) \\ &\quad +\lambda\left(H_1^2\bar{\tilde{N}}P_L\tilde{H} + H_2^1\bar{\tilde{H}}P_L\tilde{N}\right) + \text{h.c.} \quad , \end{aligned} \tag{140}$$

which leads to the interaction of the mass eigenstates

$$\mathcal{L}_{C^\pm\tilde{\chi}^0\tilde{\chi}^\pm} = -C^-\bar{\tilde{\chi}}_i^0\left[Q'^{L*}_{ij}P_L + Q'^R_{ij}P_R\right]\tilde{\chi}_j^+ + \text{h.c.} \quad , \tag{141}$$

where

$$\begin{aligned}Q'^L_{ij} &= g\cos\beta\Big[\left(-N_{i3}\sin\beta + N_{i4}\cos\beta\right)V_{j1} \\ &\qquad + \frac{1}{\sqrt{2}}\left(2s_W N_{i1} + (c_W - \frac{s_W^2}{c_W})N_{i2}\right)V_{j2}\Big] \\ &\quad -\lambda^*\sin\beta N_{i5}V_{j2} \quad , \end{aligned} \tag{142}$$

$$\begin{aligned}Q'^R_{ij} &= g\sin\beta\Big[\left(N_{i3}\cos\beta + N_{i4}\sin\beta\right)U_{j1} \\ &\qquad - \frac{1}{\sqrt{2}}\left(2s_W N_{i1} + (c_W - \frac{s_W^2}{c_W})N_{i2}\right)U_{j2}\Big] \\ &\quad -\lambda^*\cos\beta N_{i5}U_{j2} \quad . \end{aligned} \tag{143}$$

Finally, the interaction of a neutral Higgs boson and two neutralinos arises from

$$\begin{aligned}\mathcal{L}^{\text{int}}_{H\tilde{\chi}^0\tilde{\chi}^0} &= \frac{g}{\sqrt{2}c_W}\left(H_2^{2*}\bar{\tilde{Z}}P_L\left(\tilde{H}_a\cos\beta + \tilde{H}_b\sin\beta\right) - H_1^{1*}\bar{\tilde{Z}}P_L\left(-\tilde{H}_a\sin\beta + \tilde{H}_b\cos\beta\right)\right) \\ &\quad -\lambda\left(H_1^1\bar{\tilde{N}}P_L\left(-\tilde{H}_a\sin\beta + \tilde{H}_b\cos\beta\right) + H_2^2\bar{\tilde{N}}P_L\left(\tilde{H}_a\cos\beta + \tilde{H}_b\sin\beta\right)\right) \\ &\quad +2kN\bar{\tilde{N}}P_L\tilde{N} \quad + \text{h.c.} \quad . \end{aligned} \tag{144}$$

With the mass eigenstates of the Higgs bosons and neutralinos one arrives at

$$
\begin{aligned}
\mathcal{L}_{H\tilde{\chi}^0\tilde{\chi}^0} &= -\frac{1}{2} S_a \bar{\tilde{\chi}}^0_i (Q^{L''}_{aij} P_L + Q^{R''}_{aij} P_R) \tilde{\chi}^0_j \\
&\quad -\frac{i}{2} P_\alpha \bar{\tilde{\chi}}^0_i (R^{L''}_{\alpha ij} P_L + R^{R''}_{\alpha ij} P_R) \tilde{\chi}^0_j \ , \qquad (145)
\end{aligned}
$$

where

$$
\begin{aligned}
Q''^{L}_{aij} &= \frac{1}{2}\Big[ (U^S_{a1}\cos\beta + U^S_{a2}\sin\beta) \\
&\qquad \times \left( \frac{g}{c_W}(N_{i2}N^*_{j3} + N_{j2}N^*_{i3}). + \sqrt{2}\lambda(N_{i5}N^*_{j4} + N_{j5}N^*_{i4}) \right) \\
&\quad + (U^S_{a1}\sin\beta - U^S_{a2}\cos\beta) \\
&\qquad \times \left( \frac{g}{c_W}(N_{i2}N^*_{j4} + N_{j2}N^*_{i4}) - \sqrt{2}\lambda(N_{i5}N^*_{j3} + N_{j5}N^*_{i3}) \right) \Big] \\
&\quad - \sqrt{2}kU^S_{a3}(N_{i5}N^*_{j5} + N_{j5}N^*_{i5}) \ , \qquad (146) \\
Q''^{R}_{aij} &= Q''^{L*}_{aij} \ , \qquad (147) \\
R''^{L}_{\alpha ij} &= -\frac{1}{2}\Big[ (U^P_{\alpha 1}\cos\beta + U^P_{\alpha 2}\sin\beta) \\
&\qquad \times \left( \frac{g}{c_W}(N_{i2}N^*_{j3} + N_{j2}N^*_{i3}) - \sqrt{2}\lambda(N_{i5}N^*_{j4} + N_{j5}N^*_{i4}) \right) \\
&\quad + (U^P_{\alpha 1}\sin\beta - U^P_{\alpha 2}\cos\beta) \\
&\qquad \times \left( \frac{g}{c_W}(N_{i2}N^*_{j4} + N_{j2}N^*_{i4}) + \sqrt{2}\lambda(N_{i5}N^*_{j3} + N_{j5}N^*_{i3}) \right) \Big] \\
&\quad - \sqrt{2}kU^P_{\alpha 3}(N_{i5}N^*_{j5} + N_{j5}N^*_{i5}) \ , \qquad (148) \\
R''^{R}_{\alpha ij} &= -R''^{L*}_{\alpha ij} \ . \qquad (149)
\end{aligned}
$$

The corresponding Feynman rules are given in Fig. 18. All couplings of Higgs bosons to neutralinos and charginos derived in this section explicitly contain the singlet components $U^S_{a3}$, $U^P_{\alpha 3}$ of the neutral Higgs bosons or $N_{i5}$ of the neutralinos and are therefore obviously different from the MSSM couplings. They will be discussed in Sec. 5 together with some other crucial differences between the couplings of the minimal and nonminimal model.

# 4 Experimental constraints on the parameter space

The parameter space of the NMSSM and the masses of the supersymmetric particles are constrained by the results from the current high energy colliders LEP1 at CERN and Tevatron at Fermilab. In this section we reanalyze the experimental results from the negative search for neutralinos and Higgs bosons which were studied in great detail in refs. [18, 19]. A key role for the production of Higgs bosons at $e^+e^-$ colliders plays the Higgs coupling to $Z$ bosons in eqs. (66) and (70), while neutralino production at LEP1 crucially depends on the $Z\tilde{\chi}^0\tilde{\chi}^0$ coupling which is formally identical in NMSSM and MSSM and differs only

by the neutralino mixing. All those couplings are suppressed in the NMSSM if the respective neutralinos or Higgs bosons have significant singlet components. Therefore NMSSM neutralino and Higgs mass bounds are much weaker than in the minimal model. We now discuss these mass bounds and couplings in detail.

## 4.1 Constraints from Higgs search

First we consider the couplings of the scalar and pseudoscalar Higgs bosons to $Z$ bosons that are crucial for the Higgs production at future $e^+e^-$ colliders. Bounds for these couplings as a function of the Higgs masses from the experiments at LEP constrain the masses of the Higgs bosons and the parameters of the Higgs sector. In Fig. 19 we compare the couplings of a scalar Higgs boson with two $Z$ bosons in the MSSM and the NMSSM for different singlet vacuum expectation values $x$ and parameters $k$ and two values of $\tan\beta$. Radiative corrections to the Higgs masses due to top/stop loops are included with $A_t = 0$ GeV and $m_{\tilde{t}_1} = 200$ GeV, $m_{\tilde{t}_2} = 500$ GeV. All couplings are normalized with respect to the SM $\Phi ZZ$ coupling. The solid lines in Fig. 19 denote the experimental bounds from the LEP experiments [17]. Here the upper line applies if the Higgs boson decays as in the SM, the lower curve is valid if it decays invisibly. Also shown are the range of the NMSSM couplings (dashed lines for $g^{NMSSM}_{S_1ZZ}/g^{SM}_{\Phi ZZ}$, dotted for $g^{NMSSM}_{S_2ZZ}/g^{SM}_{\Phi ZZ}$) scanning over all values for the parameters $A_\lambda$ and $A_k$, and the MSSM couplings $g^{MSSM}_{hZZ}/g^{SM}_{\Phi ZZ}$ (double dashed) and $g^{MSSM}_{HZZ}/g^{SM}_{\Phi ZZ}$ (dashed dotted).

In Fig. 19 we first choose a singlet vacuum expectation value $x = 200$ GeV near those of the Higgs doublets $v = \sqrt{v_1^2 + v_2^2} = 174$ GeV. The couplings in the superpotential are fixed to typical values $\lambda = 0.8$ and $k = 0.1$ which allow light Higgs bosons with a large singlet component. The dependence on $\tan\beta$ is studied with $\tan\beta = 2$ and $\tan\beta = 10$. For the MSSM couplings the $\mu$ parameter which affects the radiative corrections is set to $\mu = \lambda x$. Furthermore, we consider in Fig. 19 the larger singlet vacuum expectation value $x = 1000$ GeV for the same set of parameters $\lambda$, $k$ and $\tan\beta$. In this case the lightest Higgs has only a small singlet component, so that we also study a smaller coupling $k = 0.01$, where for $\tan\beta = 10$ a small Higgs mass is not experimentally excluded.

First one notes the different mass ranges for the lightest and next-to-lightest Higgs bosons in the NMSSM and MSSM as well as for the different parameters sets. For $x = 200$ GeV, $\lambda = 0.8$, $k = 0.1$, $\tan\beta = 2$ and the above described choice of $A_t$ and the stop masses the theoretical upper bound for the lightest NMSSM Higgs scalar $S_1$ is about 110 GeV, while it is increased to 150 GeV for a large singlet vacuum expectation value $x = 1000$ GeV when the other parameters are unchanged. A smaller coupling $k$ lets this bounds again decrease. The second lightest Higgs scalar $S_2$ can accordingly reach higher mass regions $145\,(108)$ GeV $< m_{S_2} <$ $269\,(227)$ GeV for $x = 1000$ GeV, $\lambda = 0.8$, $k = 0.1$, $\tan\beta = 2\,(10)$, while its mass is constrained to a narrow range in the other considered scenarios. The larger value $\tan\beta = 10$ leads to a decrease of the upper mass bound for the lightest Higgs scalar of about 50 GeV. In the MSSM, the mass regions for the Higgs scalars are generally smaller for the light Higgs and larger for the heavy scalar Higgs particle.

In the MSSM, the $hZZ$ and $HZZ$ couplings for the light and heavy Higgs scalars to $Z$ or $W$ bosons are given by $\sin(\alpha - \beta)$ and $\cos(\alpha - \beta)$ relative to the SM $\Phi ZZ$

coupling, respectively. Contrary, in the NMSSM these couplings can become very small if the respective Higgs bosons have a large singlet component. Since also the coupling between the lightest scalar and pseudoscalar Higgs particles and a $Z$ boson is rather weak, as we will discuss in connection with Fig. 20, a very light or massless Higgs boson is experimentally not excluded for $x = 200$ GeV. Here the mixing of the lightest scalar NMSSM Higgs is dominated by its singlet component, while the coupling of the next-to-lightest scalar Higgs boson to gauge bosons is of the same order as in the SM for a Higgs boson of the same mass. For the larger value $\tan\beta = 10$ the squared $S_1ZZ$ coupling can be totally suppressed compared to the SM in the whole Higgs mass range.

The situation is different for $x = 1000$ GeV with $\lambda$, $k$ unchanged. Here the singlet component of the lightest Higgs scalar with a mass up to about 100 GeV practically vanishes so that the NMSSM Higgs mass bound becomes similar to that in the MSSM for $\tan\beta = 2$ or even stronger for $\tan\beta = 10$. With decreasing parameter $k$ the possible singlet component of the lightest Higgs scalar increases, so that for $k = 0.01$ and $\tan\beta = 10$ a 10-GeV Higgs scalar is not excluded.

The couplings between a scalar and pseudoscalar Higgs and a $Z$ boson in the NMSSM are shown in Fig. 20. Since for small singlet vacuum expectation values $x$ the lightest Higgs scalar has a large singlet component, while for large $x$ the light pseudoscalar Higgs boson is almost a pure singlet, they are always rather small. For our representative choice of parameters, their largest squared ratio relative to the SM $\Phi ZZ$ coupling reaches about 1 % at $x = 200$ GeV and decreases to $10^{-6}$ for $x = 1000$ GeV and $\tan\beta = 2$. Larger values of $\tan\beta$ lead to even smaller $S_{1,2}P_1Z$ couplings. As a consequence the experimental bounds from the direct search for pseudoscalar Higgs bosons produced together with a Higgs scalar at LEP do not significantly extend the excluded parameter domain or raise the Higgs mass bounds.

The constraints for the parameters $A_\lambda$ and $A_k$ of the Higgs sector and the Higgs masses are summarized in Fig. 21. Here the region above the $m_{S_1} = 0$ contour line is forbidden because the mass squared would become negative, while the domain beyond the dashed line is excluded since there exists an alternative lower minimum of the Higgs potential with vanishing vacuum expectation values. For $x = 200$ GeV a massless Higgs scalar is not excluded by the present LEP bounds while for $x = 1000$ GeV a large value of $\tan\beta$ and a small parameter $k$ are necessary for a light scalar Higgs boson. More details for the interpretation of $(A_\lambda, A_k)$ plots and excluded parameter regions due to LEP constraints can be found in ref. [19].

## 4.2 Constraints from neutralino search

Contrary to the MSSM, neutralino and Higgs sectors are strongly correlated in the NMSSM. In addition to the parameters of the Higgs sector only the gaugino mass parameters $M$ and $M'$ have to be fixed in order to determine the masses and mixings of the neutralinos. So the LEP bounds from neutralino and Higgs searches have to be combined in order to constrain the NMSSM parameter space most effectively, e. g. small singlet vacuum expectation values $x < 14$ GeV are ruled out for $\tan\beta = 2$ [19].

The consequences from the negative neutralino search at LEP for the parameter space and the neutralino masses have been studied in ref. [18]. We review our previous analysis

with the now slightly improved limits [39]

1. for new physics contributing to the total $Z$ width

$$\Delta\Gamma_Z < 23.1 \text{ MeV}, \tag{150}$$

2. for new physics contributing to the invisible $Z$ width

$$\Delta\Gamma_{\text{inv}} < 8.4 \text{ MeV}, \tag{151}$$

3. from the direct neutralino search. Here we extract the following bounds

$$B(Z \to \tilde{\chi}_1^0 \tilde{\chi}_j^0) \;<\; 2 \times 10^{-5}, \quad j = 2, \ldots, 5, \tag{152}$$

$$B(Z \to \tilde{\chi}_i^0 \tilde{\chi}_j^0) \;<\; 5 \times 10^{-5}, \quad i, j = 2, \ldots, 5. \tag{153}$$

In Fig. 22 we show the excluded parameter space due to the above constraints in the $(\lambda, k)$ plane for the same parameters as in the last section, Fig. 23 depicts the excluded regions in the $(M, x)$ plane for the same parameters as in ref. [18] in order to study the effects of the new LEP bounds. For this plots, the usual gaugino mass relation

$$M' = \frac{5}{3}\frac{g'^2}{g^2} M \simeq 0.5M \tag{154}$$

is employed.

The experimentally excluded parameter space in Fig. 22 generally becomes smaller for larger values of $x$. For larger $\tan\beta$ the excluded region from the total $Z$ width measurements also decreases, but on the other hand now the invisible $Z$ width and direct neutralino search rule out an increasing domain.

Fig. 22 shows that the improved LEP bounds only slightly changed the excluded domain. Generally, the neutralino mass bounds derived in ref. [18] are not affected by these results, e. g. a massless neutralino is still allowed in the NMSSM. In ref. [30] it is shown that a very light NMSSM neutralino cannot even be ruled out at LEP2, so that its exclusion (or detection) will be one of the challenges at a future linear $e^+e^-$ collider.

# 5 Higgs couplings

In this section we confront some particular couplings of NMSSM and MSSM, which are supposed to have some important phenomenological implications for supersymmetric processes at future particle colliders. Our analysis includes the Higgs couplings to quarks, which are generally suppressed in the NMSSM compared to the minimal model, but also the vertex factors with Higgs bosons and scalar quarks, neutralinos or charginos that can be suppressed or enhanced according to the choice of the NMSSM parameters. The Feynman rules with quarks and squarks are relevant for Higgs production via gluon fusion at proton colliders or Higgs decay into gluons or photons. For the Higgs decay into a photon pair also the Higgs-chargino-chargino couplings may be important. Furthermore,

a strong coupling of the Higgs bosons to two lightest neutralinos may enhance an invisible supersymmetric decay mode. Finally, the Higgs self-coupling may be probed at a linear $e^+e^-$ collider. So all these couplings are suited for decisive tests of the NMSSM.

In all plots in this section, the experimental constraints described in Sec. 4 are included. First we consider the couplings of the scalar neutral Higgs bosons to quarks. In the SM as well as in the MSSM and NMSSM, the Higgs-quark-antiquark couplings are proportional to the quark mass. Both the MSSM and NMSSM couplings obtain a factor depending on the Higgs mixing angles which can either suppress or enhance the vertex function. In Fig. 24 the NMSSM and MSSM couplings of the two light neutral scalar Higgs bosons with two top and two bottom quarks are plotted relative to the SM values as a function of the Higgs mass. For these plots we set $A_t = A_b = 0$. The range of the NMSSM couplings is computed by scanning over all experimentally allowed parameters $A_\lambda$ and $A_k$.

For all scenarios, the squared MSSM $ht\bar{t}$ coupling is smaller than the SM value, while the squared $hb\bar{b}$ coupling is larger. The enhancement of the vertex functions with bottom quarks and the suppression of those with top quarks becomes stronger with increasing values of $\tan\beta$. The NMSSM couplings, however, are always suppressed relative to the SM vertex functions in our scenarios where the light Higgs bosons may have significant singlet components. While the minimum of the ratio for the NMSSM couplings is of the order of $10^{-1}$ at the lower Higgs mass bound, it significantly decreases with increasing mass of the light Higgs boson. For a large Higgs mass range the Higgs couplings to quarks could practically vanish, so that the loop diagrams with quarks contributing to the Higgs-gluon-gluon and Higgs-$\gamma$-$\gamma$ couplings can be neglected. Since, however, the quark loops are generally dominant for typical scenarios with nearly degenerate squark masses [27], also the production of the lightest NMSSM Higgs boson via gluon fusion is heavily suppressed in this case. This disadvantage for the Higgs search may be partially balanced by the fact, that the quark couplings of the second lightest Higgs boson may become rather large.

We now turn to the squark couplings. As already mentioned in Sec. 3.4, the vertex functions for one Higgs boson and one left-handed and and right-handed squark explicitly depend on the Higgs singlet component, so that they may be significantly enhanced or suppressed. In Fig. 25 we plot the "average" Higgs couplings to squarks

$$\sum_{i,j=L,R} \frac{g^2_{S_a\tilde{u}_i\tilde{u}_j} + g^2_{S_a\tilde{d}_i\tilde{d}_j}}{8g^2m_W^2} = \frac{< g^2_{S_a\tilde{q}\tilde{q}} >}{g^2m_W^2} \tag{155}$$

for the lightest and next-to-lightest Higgs scalar in the NMSSM and the MSSM. Again these couplings are plotted for third generation squarks, the range for the NMSSM couplings is obtained by scanning over the allowed $(A_\lambda, A_k)$-plane with $A_t = A_b = 0$. For $x = 200$ GeV, the NMSSM couplings for the lightest Higgs scalar are weakly suppressed relative to the MSSM vertex functions, while the $S_2$ couplings are slightly enhanced. For $x = 1000$ GeV the NMSSM couplings may become rather weak in the case of small parameters $k$ or comparable to the MSSM couplings for some Higgs mass regions if the value for $k$ is increased.

The couplings of the neutral Higgs bosons to neutralinos and charginos are important in order to determine the branching ratios for the supersymmetric decay channels. The

vertex factors for charginos also affect the loop decay into two photons. The NMSSM couplings explicitly differ from the MSSM vertex functions since they depend on the singlet components of the Higgs bosons as well as of the neutralinos. Fig. 26 shows the squared SUSY Higgs couplings to neutralino and charginos relative to $g^2$ for the gaugino mass parameters $M = 100$ GeV, $M' = 0.5M$. These vertex functions crucially depend on the mixing of the Higgs bosons as well as on the mixing of the neutralinos and charginos. Due to the strong correlation of neutralino and Higgs sectors in the NMSSM, the possiblities for neutralino mixing are restricted if the Higgs parameters are fixed. While in the scenarios for the figures the light Higgs bosons have significant singlet components, the light neutralinos are not dominantly singlets. For $k = 0.1$ the singlet component of the lightest neutralino is smaller than 10 % , it increases with decreasing $k$ values. For a detailed discussion of the neutralino singlet component as a function of the NMSSM parameters see ref. [30]. Due to the neutralino and chargino mixing the NMSSM couplings of the lightest neutral scalar Higgs boson to neutralinos and charginos are generally somewhat smaller in our scenarios than the MSSM couplings. As for the previously discussed couplings, there again exists a Higgs mass range where the NMSSM couplings could be heavily suppressed.

Finally we compare the trilinear Higgs self-couplings of NMSSM, MSSM and SM. In the SM the vertex function is directly proportional to the squared Higgs mass $m_\Phi^2$

$$g_{\Phi\Phi\Phi} = \frac{3}{2}\frac{m_\Phi^2}{m_W}, \tag{156}$$

while the MSSM couplings are suppressed [11, 28]:

$$\begin{aligned}
g_{hhh} &= \frac{-3}{2}\frac{gm_z}{\cos\theta_W}\cos 2\alpha\sin(\alpha+\beta), && (157)\\
g_{HHH} &= \frac{-3}{2}\frac{gm_z}{\cos\theta_W}\cos 2\alpha\cos(\alpha+\beta), && (158)\\
g_{hHH} &= \frac{g}{2}\frac{m_z}{\cos\theta_W}\left(2\sin 2\alpha\cos(\alpha+\beta)+\cos 2\alpha\sin(\alpha+\beta)\right), && (159)\\
g_{Hhh} &= \frac{g}{2}\frac{m_z}{\cos\theta_W}\left(2\sin 2\alpha\sin(\alpha+\beta)-\cos 2\alpha\cos(\alpha+\beta)\right), && (160)\\
g_{hAA} &= \frac{-g}{2}\frac{m_z}{\cos\theta_W}\cos 2\beta\sin(\alpha+\beta), && (161)\\
g_{HAA} &= \frac{-g}{2}\frac{m_z}{\cos\theta_W}\cos 2\beta\cos(\alpha+\beta). && (162)
\end{aligned}$$

In the NMSSM, however, the trilinear Higgs self-couplings contain terms proportional to the singlet vacuum expectation value $x$ and the parameters $A_\lambda$ and $A_k$. Since these parameters can in principle become as large as some TeV even for a very small Higgs mass, the Higgs self-couplings may become very strong. We show in Fig. 27 the trilinear self-coupling of the lightest Higgs scalar and also the coupling between two lightest pseudoscalar Higgs particles and the lightest scalar Higgs boson in the NMSSM and MSSM. Again, the couplings are normalized with respect to the self-coupling in the SM. While the MSSM self-couplings are always smaller than the SM couplings with a Higgs boson of the same mass, the situation is completely different in the NMSSM. Here large $A_\lambda$ and

$A_k$ values lead to an increase of the squared self-couplings by a factor up to $10^5$. This fact could be a crucial key for distinguishing between minimal and nonminimal model, if one probes the trilinear Higgs vertex e. g. in processes as $e^+e^- \to S_1S_1Z$ at a future linear collider [31].

# 6 Conclusion

We have presented a complete set of Feynman rules in the NMSSM which can be used as starting point for all calculations of NMSSM processes. The couplings of NMSSM and MSSM mainly differ by the mixing of the neutralinos and Higgs bosons, which contain an additional singlet component in the NMSSM. The following NMSSM vertex functions depend only on the doublet components of the Higgs bosons and are therefore generally suppressed in the NMSSM compared to the minimal model:

- the trilinear and quartic Higgs couplings to gauge bosons. Contrary to the MSSM, however, the vertex functions with two different neutral scalar Higgs bosons and two gauge bosons do not vanish in the NMSSM.
- the trilinear Higgs couplings to quarks and leptons.
- the trilinear or quartic Higgs couplings to two left-handed or two right-handed scalar quarks or leptons.

Moreover, some Feynman rules depend explicitly on the singlet components of the Higgs bosons. These NMSSM vertex functions which may be suppressed or enhanced compared to the MSSM are

- the trilinear and quartic Higgs couplings to one left-handed and one right-handed scalar quark. The quartic Higgs couplings to one left-handed and one right-handed scalar quark or lepton even vanish in the MSSM.
- the trilinear and quartic Higgs self-couplings. These vertex functions probably exhibit the most significant differences between the minimal and nonminimal supersymmetric model. Unfortunately, they cannot be fully tested until new powerful colliders start operating.
- the trilinear coupling of one Higgs boson to neutralinos or charginos.

We have demonstrated some fundamental differences between the SM, the minimal and the nonminimal supersymmetric model by comparing the Higgs couplings to gauge bosons, quarks, scalar quarks, neutralinos and charginos as well as the trilinear Higgs self-couplings. The vertex functions with gauge bosons are generally reduced in both supersymmetric models compared to the SM. The MSSM quark coupling may be suppressed or enhanced relative to the SM, but is always enhanced compared to the NMSSM. The ratio between the couplings with scalar quarks, neutralinos, charginos as well as the Higgs self-couplings of MSSM and NMSSM is not determined, it depends on the choice for the parameters and therefore on the Higgs and neutralino masses and mixings.

We also have reanalyzed the excluded parameter space by applying the derived Feynman rules to the experimental bounds from the so far negative search for Higgs bosons and neutralinos. Especially very light neutralinos and Higgs bosons are not excluded in the NMSSM.

Until now, there is no experimental evidence against supersymmetry – the by far largest part of the parameter space of NMSSM and MSSM is compatible with all experimental results. In order to reveal the nature of new physics all phenomenological implications of the different supersymmetric models have to be studied by computing relevant cross section and decay rates and by providing Monte-Carlo Simulations for the present detectors. The discussion of concrete supersymmetric processes was beyond the scope of this paper. The derived Feynman rules of the NMSSM represent an indispensable prerequisite for this task clearing the way for further efforts needed to verify or to exclude supersymmetry at the next generation of particle colliders.


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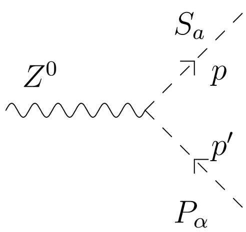
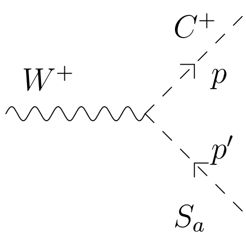
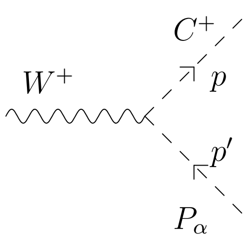



$$\frac{g}{2\cos\theta_W}(U^S_{a1}U^P_{\alpha 1} - U^S_{a2}U^P_{\alpha 2})(p+p')^\mu$$

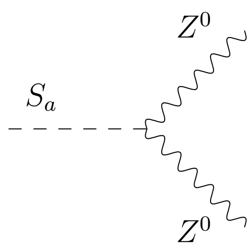



$$\frac{ig}{2}(\sin\beta U^S_{a1} - \cos\beta U^S_{a2})(p+p')^\mu$$

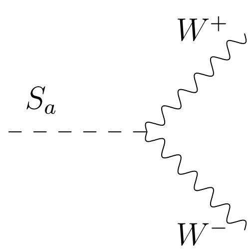



$$\frac{g}{2}(\sin\beta U^P_{\alpha 1} + \cos\beta U^P_{\alpha 2})(p+p')^\mu$$

Figure 1: Feynman rules for the $ZS_aP_\alpha$, $W^+C^+S_a$ and $W^+C^+P_\alpha$ vertices.

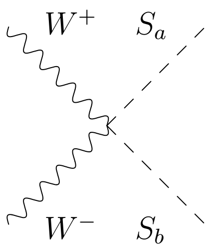



$$\frac{igm_Z}{\cos\theta_W}(\cos\beta U^S_{a1} + \sin\beta U^S_{a2})g^{\mu\nu}$$

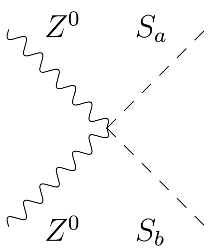



$$igm_W(\cos\beta U^S_{a1} + \sin\beta U^S_{a2})g^{\mu\nu}$$

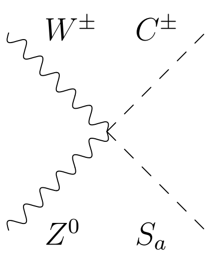
Figure 2: Feynman rules for the vertices with two gauge bosons and one neutral scalar Higgs boson.



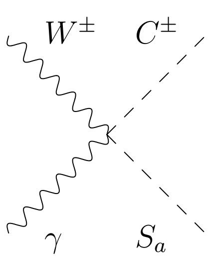

$$\frac{ig^2}{2}(U^S_{a1}U^S_{b1}+U^S_{a2}U^S_{b2})g^{\mu\nu}$$




$$\frac{ig^2}{2\cos^2\theta_W}(U^S_{a1}U^S_{b1}+U^S_{a2}U^S_{b2})g^{\mu\nu}$$

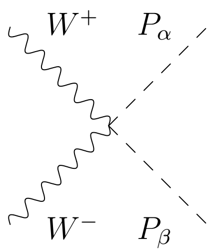



$$\frac{ig^2\sin^2\theta_W}{2\cos^2\theta_W}g^{\mu\nu}(U^S_{a1}\sin\beta-U^S_{a2}\cos\beta)$$

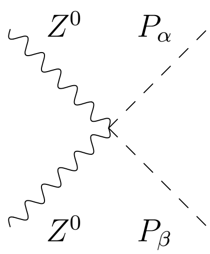



$$\frac{-ieg}{2}g^{\mu\nu}(U^S_{a1}\sin\beta-U^S_{a2}\cos\beta)$$

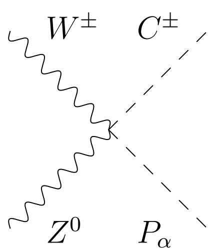
Figure 3: Feynman rules for the quartic couplings of scalar Higgs bosons to gauge bosons.



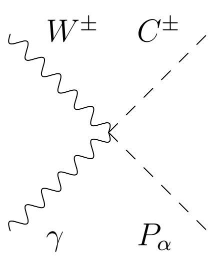

$$\frac{ig^2}{2}(U^P_{\alpha 1}U^P_{\beta 1} + U^P_{\alpha 2}U^P_{\beta 2})g^{\mu\nu}$$




$$\frac{ig^2}{2\cos^2\theta_W}(U^P_{\alpha 1}U^P_{\beta 1} + U^P_{\alpha 2}U^P_{\beta 2})g^{\mu\nu}$$

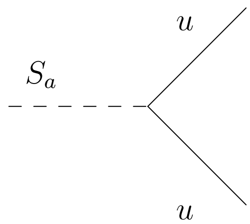



$$\pm\frac{g^2\sin^2\theta_W}{2\cos^2\theta_W}g^{\mu\nu}(U^P_{\alpha 1}\sin\beta + U^P_{\alpha 2}\cos\beta)$$

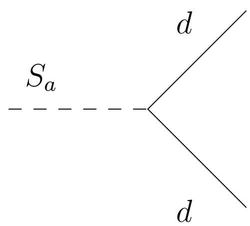



$$\mp\frac{eg}{2}g^{\mu\nu}(U^P_{\alpha 1}\sin\beta + U^P_{\alpha 2}\cos\beta)$$

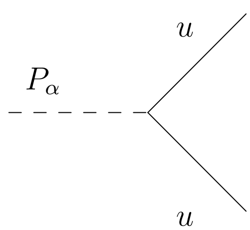
Figure 4: Feynman rules for the quartic couplings of pseudoscalar Higgs bosons to gauge bosons.



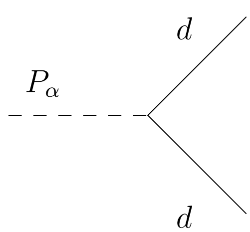

$$\frac{-igm_u}{2m_W \sin\beta} U^S_{a2}$$




$$\frac{-igm_d}{2m_W \cos\beta} U^S_{a1}$$

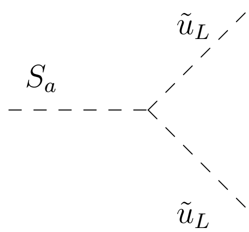



$$\frac{-gm_u}{2m_W \sin\beta} U^P_{\alpha 2}\gamma_5$$

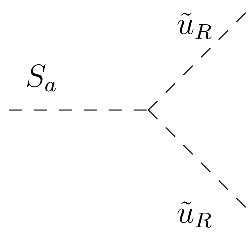



$$\frac{-gm_d}{2m_W \cos\beta} U^P_{\alpha 1}\gamma_5$$

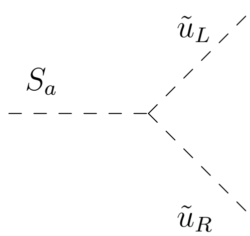
Figure 5: Feynman rules for the couplings of neutral Higgs bosons to quarks.




$$\frac{-igm_u^2}{m_W \sin\beta} U^S_{a2} + \frac{ig}{2}\frac{m_Z}{\cos\theta_W}(1-2e_u\sin^2\theta_W)(U^S_{a2}\sin\beta - U^S_{a1}\cos\beta)$$

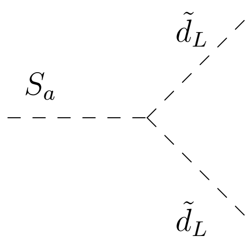



$$\frac{-igm_u^2}{m_W \sin\beta} U^S_{a2} + igm_W e_u \tan^2\theta_W (U^S_{a2}\sin\beta - U^S_{a1}\cos\beta)$$

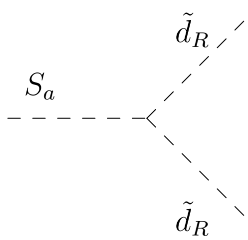



$$\frac{-igm_u}{2m_W \sin\beta}(\lambda(v_1 U^S_{a3} + xU^S_{a1}) + A_U U^S_{a2})$$

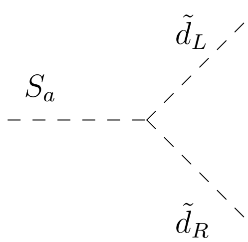
Figure 6: Feynman rules for the trilinear vertices with one neutral scalar Higgs boson and two scalar up-type quarks.




$$\frac{-igm_d^2}{m_W\cos\beta}U^S_{a1}$$
$$-\frac{ig}{2}\frac{m_Z}{\cos\theta_W}(1+2e_d\sin^2\theta_W)(U^S_{a2}\sin\beta-U^S_{a1}\cos\beta)$$

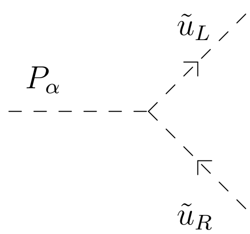



$$\frac{-igm_d^2}{m_W\cos\beta}U^S_{a1}$$
$$+igm_We_d\tan^2\theta_W(U^S_{a2}\sin\beta-U^S_{a1}\cos\beta)$$

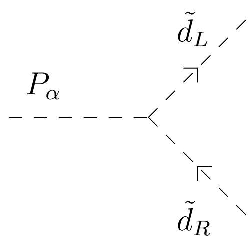



$$\frac{-igm_d}{2m_W\cos\beta}(\lambda(v_2U^S_{a3}+xU^S_{a2})+A_DU^S_{a1})$$

Figure 7: Feynman rules for the trilinear vertices with one neutral scalar Higgs boson and two scalar down-type quarks.

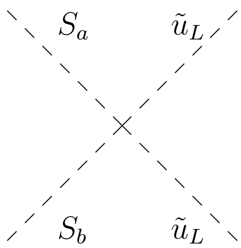



$$\frac{gm_u}{2m_W \sin\beta}(\lambda(v_1 U^P_{\alpha 3} + x U^P_{\alpha 1}) - A_U U^P_{\alpha 2})$$

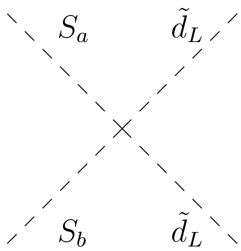



$$\frac{gm_d}{2m_W \cos\beta}(\lambda(v_2 U^P_{\alpha 3} + x U^P_{\alpha 2}) - A_D U^P_{\alpha 1})$$

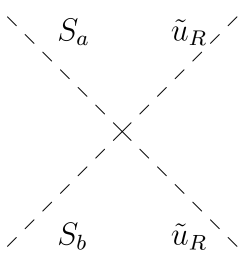
Figure 8: Feynman rules for the trilinear vertices with one neutral pseudoscalar Higgs boson and two scalar quarks.



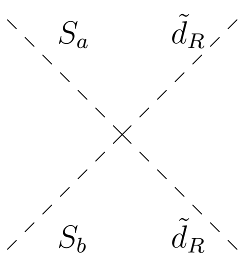

$$\frac{ig^2}{4}\left[\left(\frac{1}{\cos^2\theta_W} - 2e_u \tan^2\theta_W\right)(U^S_{a2}U^S_{b2} - U^S_{a1}U^S_{b1}) \right.$$
$$\left. -2\frac{m_u^2}{m_W^2 \sin^2\beta} U^S_{a2}U^S_{b2} \quad \right]$$




$$\frac{-ig^2}{4}\left[\left(\frac{1}{\cos^2\theta_W} + 2e_d \tan^2\theta_W\right)(U^S_{a2}U^S_{b2} - U^S_{a1}U^S_{b1}) \right.$$
$$\left. +2\frac{m_d^2}{m_W^2 \cos^2\beta} U^S_{a1}U^S_{b1} \quad \right]$$

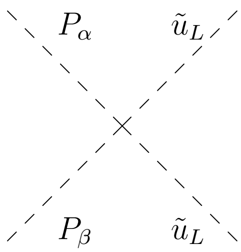



$$\frac{ig^2}{2}[e_u \tan^2\theta_W (U^S_{a2}U^S_{b2} - U^S_{a1}U^S_{b1})$$
$$-\frac{m_u^2}{m_W^2 \sin^2\beta} U^S_{a2}U^S_{b2} \quad ]$$

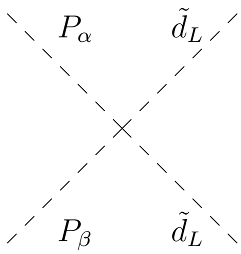



$$\frac{ig^2}{2}[e_d \tan^2\theta_W (U^S_{a2}U^S_{b2} - U^S_{a1}U^S_{b1})$$
$$-\frac{m_d^2}{m_W^2 \cos^2\beta} U^S_{a1}U^S_{b1} \quad ]$$

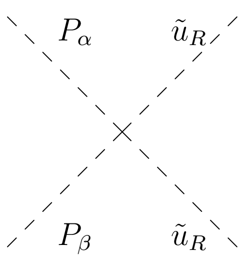
Figure 9: Feynman rules for the quartic interactions of two scalar Higgs bosons and two left-handed or two right-handed scalar quarks.



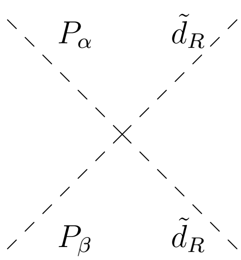

$$\frac{ig^2}{4}\left[\left(\frac{1}{\cos^2\theta_W} - 2e_u \tan^2\theta_W\right)(U^P_{\alpha 2}U^P_{\beta 2} - U^P_{\alpha 1}U^P_{\beta 1}) \right.$$
$$\left. -2\frac{m_u^2}{m_W^2 \sin^2\beta} U^P_{\alpha 2}U^P_{\beta 2} \quad \right]$$




$$\frac{ig^2}{4}\left[\left(\frac{1}{\cos^2\theta_W} + 2e_d \tan^2\theta_W\right)(U^P_{\alpha 2}U^P_{\beta 2} - U^P_{\alpha 1}U^P_{\beta 1}) \right.$$
$$\left. -2\frac{m_d^2}{m_W^2 \cos^2\beta} U^P_{\alpha 1}U^P_{\beta 1} \quad \right]$$

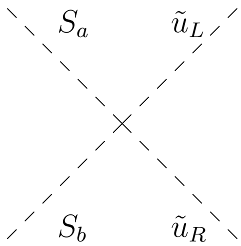



$$\frac{ig^2}{2}[e_u \tan^2\theta_W (U^P_{\alpha 2}U^P_{\beta 2} - U^P_{\alpha 1}U^P_{\beta 1})$$
$$-\frac{m_u^2}{m_W^2 \sin^2\beta} U^P_{\alpha 2}U^P_{\beta 2} \quad ]$$

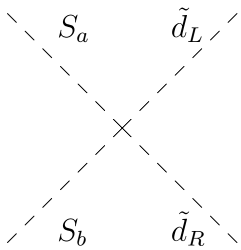



$$\frac{ig^2}{2}[e_d \tan^2\theta_W (U^P_{\alpha 2}U^P_{\beta 2} - U^P_{\alpha 1}U^P_{\beta 1})$$
$$-\frac{m_d^2}{m_W^2 \cos^2\beta} U^P_{\alpha 1}U^P_{\beta 1} \quad ]$$

Figure 10: Feynman rules for the quartic interactions of two pseudoscalar Higgs bosons and two left-handed or two right-handed scalar quarks.

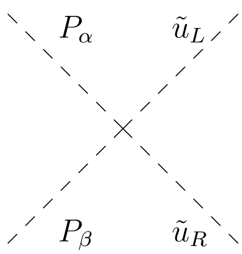



$$\frac{-ig\lambda m_u}{2\sqrt{2}m_W \sin\beta}(U^S_{a1}U^S_{b3} + U^S_{a3}U^S_{b1})$$

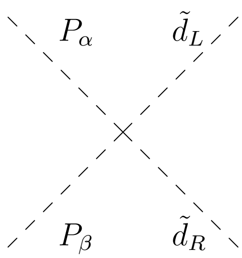



$$\frac{-ig\lambda m_d}{2\sqrt{2}m_W \cos\beta}(U^S_{a2}U^S_{b3} + U^S_{a3}U^S_{b2})$$

Figure 11: Feynman rules for the quartic interactions of two scalar Higgs bosons and one left-handed and one right-handed scalar quark.

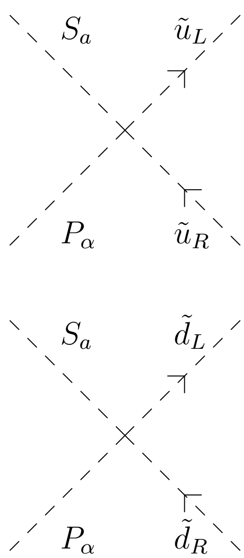



$$\frac{ig\lambda m_u}{2\sqrt{2}m_W \sin\beta}(U^P_{\alpha1}U^P_{\beta3} + U^P_{\alpha3}U^P_{\beta1})$$




$$\frac{ig\lambda m_d}{2\sqrt{2}m_W \cos\beta}(U^P_{\alpha2}U^P_{\beta3} + U^P_{\alpha3}U^P_{\beta2})$$

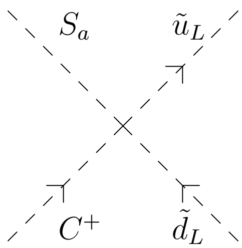
Figure 12: Feynman rules for the quartic interactions of two pseudoscalar Higgs bosons and one left-handed and one right-handed scalar quark.



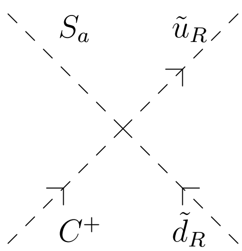

$$\frac{g\lambda m_u}{2\sqrt{2}m_W \sin\beta}(U^S_{a1}U^P_{\alpha 3} + U^S_{a3}U^P_{\alpha 1})$$

$$\frac{g\lambda m_d}{2\sqrt{2}m_W \cos\beta}(U^S_{a2}U^P_{\alpha 3} + U^S_{a3}U^P_{\alpha 2})$$

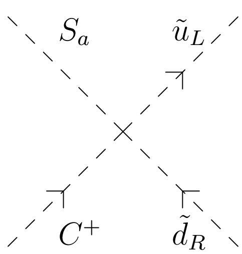
Figure 13: Feynman rules for the quartic interactions of one scalar and one pseudoscalar Higgs boson and two scalar quarks.



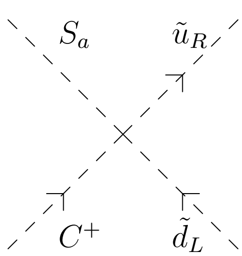

$$\frac{-ig^2}{2\sqrt{2}}\left(U^S_{a1}\sin\beta + U^S_{a2}\cos\beta - \frac{m_u^2}{m_W^2}\frac{\cos\beta}{\sin^2\beta}U^S_{a2} - \frac{m_d^2}{m_W^2}\frac{\sin\beta}{\cos^2\beta}U^S_{a1}\right)$$




$$\frac{ig^2 m_u m_d}{\sqrt{2}m_W^2\sin 2\beta}\left(U^S_{a2}\sin\beta + U^S_{a1}\cos\beta\right)$$

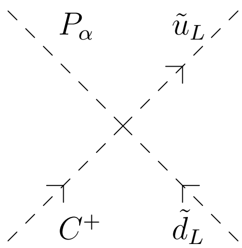



$$\frac{-ig\lambda m_d}{2m_W}U^S_{a3}$$

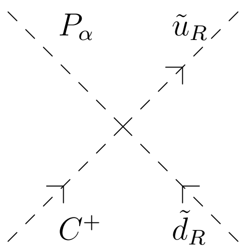



$$\frac{-ig\lambda m_u}{2m_W}U^S_{a3}$$

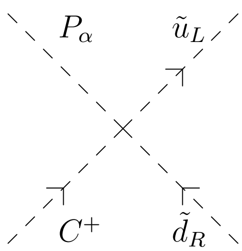
Figure 14: Feynman rules for the quartic interactions of one scalar and one charged Higgs boson and two scalar quarks.



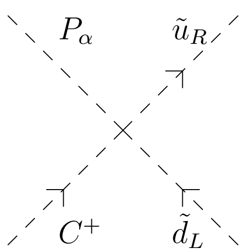

$$\frac{g^2}{2\sqrt{2}}\Big(U^P_{\alpha 1}\sin\beta - U^P_{\alpha 2}\cos\beta + \frac{m_u^2}{m_W^2}\frac{\cos\beta}{\sin^2\beta}U^P_{\alpha 2} - \frac{m_d^2}{m_W^2}\frac{\sin\beta}{\cos^2\beta}U^P_{\alpha 1}\Big)$$




$$\frac{g^2 m_u m_d}{\sqrt{2}m_W^2\sin 2\beta}(U^P_{\alpha 2}\sin\beta - U^P_{\alpha 1}\cos\beta)$$

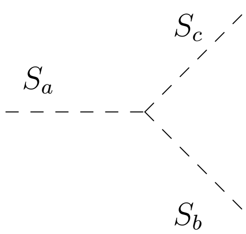



$$\frac{g\lambda m_d}{2m_W}U^P_{\alpha 3}$$

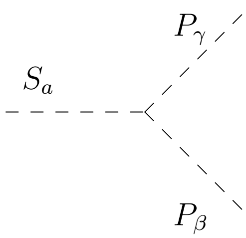



$$-\frac{g\lambda m_u}{2m_W}U^P_{\alpha 3}$$

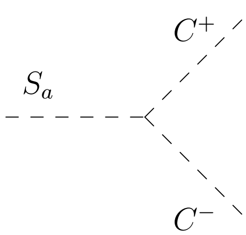
Figure 15: Feynman rules for the quartic interactions of one pseudoscalar and one charged Higgs boson and two scalar quarks.




$$
\begin{aligned}
&-\tfrac{3}{2} i \tfrac{g^2+g'^2}{\sqrt{2}} (v_1 U^S_{a1} U^S_{b1} U^S_{c1} + v_2 U^S_{a2} U^S_{b2} U^S_{c2}) \\
&+i \left( \tfrac{g^2+g'^2}{2\sqrt{2}} - \sqrt{2}\lambda^2 \right) v_1 (U^S_{a1} U^S_{b2} U^S_{c2} + U^S_{a2} U^S_{b1} U^S_{c2} + U^S_{a2} U^S_{b2} U^S_{c1}) \\
&+i \left( \tfrac{g^2+g'^2}{2\sqrt{2}} - \sqrt{2}\lambda^2 \right) v_2 (U^S_{a1} U^S_{b1} U^S_{c2} + U^S_{a1} U^S_{b2} U^S_{c1} + U^S_{a2} U^S_{b1} U^S_{c1}) \\
&+\sqrt{2} i (\lambda k v_2 - \lambda^2 v_1)(U^S_{a1} U^S_{b3} U^S_{c3} + U^S_{a3} U^S_{b1} U^S_{c3} + U^S_{a3} U^S_{b3} U^S_{c1}) \\
&+\sqrt{2} i (\lambda k v_1 - \lambda^2 v_2)(U^S_{a2} U^S_{b3} U^S_{c3} + U^S_{a3} U^S_{b2} U^S_{c3} + U^S_{a3} U^S_{b3} U^S_{c2}) \\
&-\sqrt{2} i \lambda^2 x (U^S_{a1} U^S_{b1} U^S_{c3} + U^S_{a1} U^S_{b3} U^S_{c1} + U^S_{a3} U^S_{b1} U^S_{c1} \\
&\qquad + U^S_{a2} U^S_{b2} U^S_{c3} + U^S_{a2} U^S_{b3} U^S_{c2} + U^S_{a3} U^S_{b2} U^S_{c2}) \\
&+i\lambda \left( \tfrac{A_\lambda}{\sqrt{2}} + \sqrt{2} k x \right) (U^S_{a1} U^S_{b2} U^S_{c3} + U^S_{a1} U^S_{b3} U^S_{c2} + U^S_{a2} U^S_{b1} U^S_{c3} \\
&\qquad + U^S_{a2} U^S_{b3} U^S_{c1} + U^S_{a3} U^S_{b1} U^S_{c2} + U^S_{a3} U^S_{b2} U^S_{c1}) \\
&+i(\sqrt{2} k A_k - 6\sqrt{2} k^2) U^S_{a3} U^S_{b3} U^S_{c3}
\end{aligned}
$$

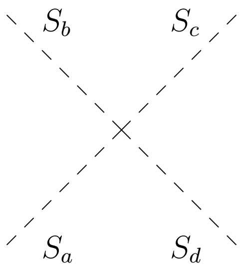



$$
\begin{aligned}
&-i \tfrac{g^2+g'^2}{2\sqrt{2}} \left( v_1 U^S_{a1} U^P_{\beta 1} U^P_{\gamma 1} + v_2 U^S_{a2} U^P_{\beta 2} U^P_{\gamma 2} \right) \\
&+ \left( i \tfrac{g^2+g'^2}{2\sqrt{2}} - \sqrt{2}\lambda^2 \right) \left( v_1 U^S_{a1} U^P_{\beta 2} U^P_{\gamma 2} + v_2 U^S_{a2} U^P_{\beta 1} U^P_{\gamma 1} \right) \\
&-\sqrt{2} i (\lambda k v_1 + \lambda^2 v_2) U^S_{a2} U^P_{\beta 3} U^P_{\gamma 3} \\
&-\sqrt{2} i (\lambda k v_2 + \lambda^2 v_1) U^S_{a1} U^P_{\beta 3} U^P_{\gamma 3} \\
&-\sqrt{2} i \lambda^2 x U^S_{a3} (U^P_{\beta 1} U^P_{\gamma 1} + U^P_{\beta 2} U^P_{\gamma 2}) \\
&-i \left( 2\sqrt{2} k^2 x + \sqrt{2} k A_k \right) U^S_{a3} U^P_{\beta 3} U^P_{\gamma 3} \\
&+\sqrt{2} i \lambda k U^S_{a3} \left( v_1 (U^P_{\beta 2} U^P_{\gamma 3} + U^P_{\beta 3} U^P_{\gamma 2}) + v_2 (U^P_{\beta 1} U^P_{\gamma 3} + U^P_{\beta 3} U^P_{\gamma 1}) \right) \\
&+i \left( \sqrt{2} \lambda k x - \tfrac{\lambda A_\lambda}{\sqrt{2}} \right) \left( U^S_{a1} (U^P_{\beta 2} U^P_{\gamma 3} + U^P_{\beta 3} U^P_{\gamma 2}) \right. \\
&\qquad\qquad \left. + U^S_{a2} (U^P_{\beta 1} U^P_{\gamma 3} + U^P_{\beta 3} U^P_{\gamma 1} \right) \\
&-i \left( \sqrt{2} \lambda k x + \tfrac{\lambda A_\lambda}{\sqrt{2}} \right) U^S_{a3} (U^P_{\beta 1} U^P_{\gamma 2} + U^P_{\beta 2} U^P_{\gamma 1})
\end{aligned}
$$

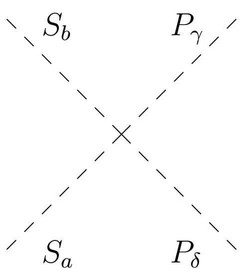



$$
\begin{aligned}
&-i g m_W (U^S_{a1} \cos\beta + U^S_{a2} \sin\beta) \\
&-i \tfrac{g m_Z}{2\cos\theta_W} \left( U^S_{a2} \sin\beta - U^S_{a1} \cos\beta \right) \cos 2\beta \\
&+i \tfrac{\lambda^2}{\sqrt{2}} \left( v_1 U^S_{a2} + v_2 U^S_{a1} \right) \sin 2\beta \\
&-\tfrac{i}{\sqrt{2}} \lambda U^S_{a3} \left[ (2kx + A_\lambda) \sin 2\beta + 2\lambda x \right]
\end{aligned}
$$

Figure 16: Feynman rules for the trilinear self-interactions of the Higgs bosons.

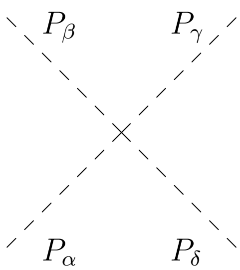



$$
\begin{aligned}
&-i\tfrac{3}{4}(g^2+g'^2)(U^S_{a1}U^S_{b1}U^S_{c1}U^S_{d1}+U^S_{a2}U^S_{b2}U^S_{c2}U^S_{d2})\\
&+i\left(\tfrac{1}{4}(g^2+g'^2)-\lambda^2\right)\\
&\qquad(U^S_{a1}U^S_{b1}U^S_{c2}U^S_{d2}+U^S_{a1}U^S_{b2}U^S_{c1}U^S_{d2}+U^S_{a1}U^S_{b2}U^S_{c2}U^S_{d1}\\
&\qquad\ +U^S_{a2}U^S_{b1}U^S_{c1}U^S_{d2}+U^S_{a2}U^S_{b1}U^S_{c2}U^S_{d1}+U^S_{a2}U^S_{b2}U^S_{c1}U^S_{d1})\\
&-i\lambda^2(U^S_{a1}U^S_{b1}U^S_{c3}U^S_{d3}+U^S_{a1}U^S_{b3}U^S_{c1}U^S_{d3}+U^S_{a1}U^S_{b3}U^S_{c3}U^S_{d1}\\
&\qquad\ +U^S_{a3}U^S_{b1}U^S_{c1}U^S_{d3}+U^S_{a3}U^S_{b1}U^S_{c3}U^S_{d1}+U^S_{a3}U^S_{b3}U^S_{c1}U^S_{d1}\\
&\qquad\ +U^S_{a2}U^S_{b2}U^S_{c3}U^S_{d3}+U^S_{a2}U^S_{b3}U^S_{c2}U^S_{d3}+U^S_{a2}U^S_{b3}U^S_{c3}U^S_{d2}\\
&\qquad\ +U^S_{a3}U^S_{b2}U^S_{c2}U^S_{d3}+U^S_{a3}U^S_{b2}U^S_{c3}U^S_{d2}+U^S_{a3}U^S_{b3}U^S_{c2}U^S_{d2})\\
&-6ik^2U^S_{a3}U^S_{b3}U^S_{c3}U^S_{d3}\\
&-i\lambda k(U^S_{a1}U^S_{b2}U^S_{c3}U^S_{d3}+U^S_{a1}U^S_{b3}U^S_{c2}U^S_{d3}+U^S_{a1}U^S_{b3}U^S_{c3}U^S_{d2}\\
&\qquad\ +U^S_{a2}U^S_{b1}U^S_{c3}U^S_{d3}+U^S_{a2}U^S_{b3}U^S_{c1}U^S_{d3}+U^S_{a2}U^S_{b3}U^S_{c3}U^S_{d1}\\
&\qquad\ +U^S_{a3}U^S_{b1}U^S_{c2}U^S_{d3}+U^S_{a3}U^S_{b1}U^S_{c3}U^S_{d2}+U^S_{a3}U^S_{b2}U^S_{c1}U^S_{d3}\\
&\qquad\ +U^S_{a3}U^S_{b2}U^S_{c3}U^S_{d1}+U^S_{a3}U^S_{b3}U^S_{c1}U^S_{d2}+U^S_{a3}U^S_{b3}U^S_{c2}U^S_{d1})
\end{aligned}
$$

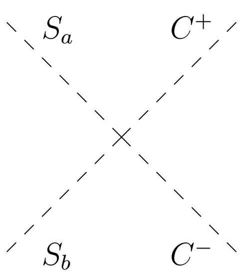



$$
\begin{aligned}
&-i\tfrac{1}{4}(g^2+g'^2)(U^S_{a1}U^S_{b1}U^P_{\gamma1}U^P_{\delta1}+U^S_{a2}U^S_{b2}U^P_{\gamma2}U^P_{\delta2})\\
&+i\left(\tfrac{1}{4}(g^2+g'^2)-\lambda^2\right)(U^S_{a1}U^S_{b1}U^P_{\gamma2}U^P_{\delta2}+U^S_{a2}U^S_{b2}U^P_{\gamma1}U^P_{\delta1})\\
&-i\lambda^2(U^S_{a1}U^S_{b1}U^P_{\gamma3}U^P_{\delta3}+U^S_{a3}U^S_{b3}U^P_{\gamma1}U^P_{\delta1}\\
&\qquad\ +U^S_{a2}U^S_{b2}U^P_{\gamma3}U^P_{\delta3}+U^S_{a3}U^S_{b3}U^P_{\gamma2}U^P_{\delta2})\\
&-2ik^2U^S_{a3}U^S_{b3}U^P_{\gamma3}U^S_{\delta3}\\
&-i\lambda k\Big((U^S_{a1}U^S_{b2}+U^S_{a2}U^S_{b1})U^P_{\gamma3}U^P_{\delta3}+U^S_{a3}U^S_{b3}(U^P_{\gamma1}U^P_{\delta2}+U^P_{\gamma2}U^P_{\delta1})\\
&\qquad\ -U^S_{a1}U^S_{b3}U^P_{\gamma2}U^P_{\delta3}-U^S_{a3}U^S_{b1}U^P_{\gamma2}U^P_{\delta3}\\
&\qquad\ -U^S_{a1}U^S_{b3}U^P_{\gamma3}U^P_{\delta2}-U^S_{a3}U^S_{b1}U^P_{\gamma3}U^P_{\delta2}\\
&\qquad\ -U^S_{a2}U^S_{b3}U^P_{\gamma1}U^P_{\delta3}-U^S_{a3}U^S_{b2}U^P_{\gamma1}U^P_{\delta3}\\
&\qquad\ -U^S_{a2}U^S_{b3}U^P_{\gamma3}U^P_{\delta1}-U^S_{a3}U^S_{b2}U^P_{\gamma3}U^P_{\delta1}\Big)
\end{aligned}
$$

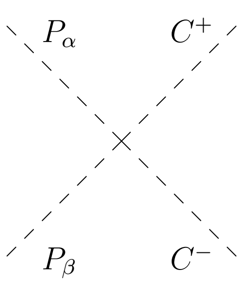
Figure 17: Feynman rules for the quartic self-interactions of the Higgs bosons.




$$
\begin{aligned}
&-i\tfrac{3}{4}(g^2+g'^2)(U^P_{\alpha1}U^P_{\beta1}U^P_{\gamma1}U^P_{\delta1}+U^P_{\alpha2}U^P_{\beta2}U^P_{\gamma2}U^P_{\delta2})\\
&+i\left(\tfrac{1}{4}(g^2+g'^2)-\lambda^2\right)\\
&\qquad\quad(U^P_{\alpha1}U^P_{\beta1}U^P_{\gamma2}U^P_{\delta2}+U^P_{\alpha1}U^P_{\beta2}U^P_{\gamma1}U^P_{\delta2}+U^P_{\alpha1}U^P_{\beta2}U^P_{\gamma2}U^P_{\delta1}\\
&\qquad\quad+U^P_{\alpha2}U^P_{\beta1}U^P_{\gamma1}U^P_{\delta2}+U^P_{\alpha2}U^P_{\beta1}U^P_{\gamma2}U^P_{\delta1}+U^P_{\alpha2}U^P_{\beta2}U^P_{\gamma1}U^P_{\delta1})\\
&-i\lambda^2(U^P_{\alpha1}U^P_{\beta1}U^P_{\gamma3}U^P_{\delta3}+U^P_{\alpha1}U^P_{\beta3}U^P_{\gamma1}U^P_{\delta3}+U^P_{\alpha1}U^P_{\beta3}U^P_{\gamma3}U^P_{\delta1}\\
&\qquad\quad+U^P_{\alpha3}U^P_{\beta1}U^P_{\gamma1}U^P_{\delta3}+U^P_{\alpha3}U^P_{\beta1}U^P_{\gamma3}U^P_{\delta1}+U^P_{\alpha3}U^P_{\beta3}U^P_{\gamma1}U^P_{\delta1}\\
&\qquad\quad+U^P_{\alpha2}U^P_{\beta2}U^P_{\gamma3}U^P_{\delta3}+U^P_{\alpha2}U^P_{\beta3}U^P_{\gamma2}U^P_{\delta3}+U^P_{\alpha2}U^P_{\beta3}U^P_{\gamma3}U^P_{\delta2}\\
&\qquad\quad+U^P_{\alpha3}U^P_{\beta2}U^P_{\gamma2}U^P_{\delta3}+U^P_{\alpha3}U^P_{\beta2}U^P_{\gamma3}U^P_{\delta2}+U^P_{\alpha3}U^P_{\beta3}U^P_{\gamma2}U^P_{\delta2})\\
&-6ik^2U^P_{\alpha3}U^P_{\beta3}U^P_{\gamma3}U^P_{\delta3}\\
&-i\lambda k(U^P_{\alpha1}U^P_{\beta2}U^P_{\gamma3}U^P_{\delta3}+U^P_{\alpha1}U^P_{\beta3}U^P_{\gamma2}U^P_{\delta3}+U^P_{\alpha1}U^P_{\beta3}U^P_{\gamma3}U^P_{\delta2}\\
&\qquad\quad+U^P_{\alpha2}U^P_{\beta1}U^P_{\gamma3}U^P_{\delta3}+U^P_{\alpha2}U^P_{\beta3}U^P_{\gamma1}U^P_{\delta3}+U^P_{\alpha2}U^P_{\beta3}U^P_{\gamma3}U^P_{\delta1}\\
&\qquad\quad+U^P_{\alpha3}U^P_{\beta1}U^P_{\gamma2}U^P_{\delta3}+U^P_{\alpha3}U^P_{\beta1}U^P_{\gamma3}U^P_{\delta2}+U^P_{\alpha3}U^P_{\beta2}U^P_{\gamma1}U^P_{\delta3}\\
&\qquad\quad+U^P_{\alpha3}U^P_{\beta2}U^P_{\gamma3}U^P_{\delta1}+U^P_{\alpha3}U^P_{\beta3}U^P_{\gamma1}U^P_{\delta2}+U^P_{\alpha3}U^P_{\beta3}U^P_{\gamma2}U^P_{\delta1})
\end{aligned}
$$

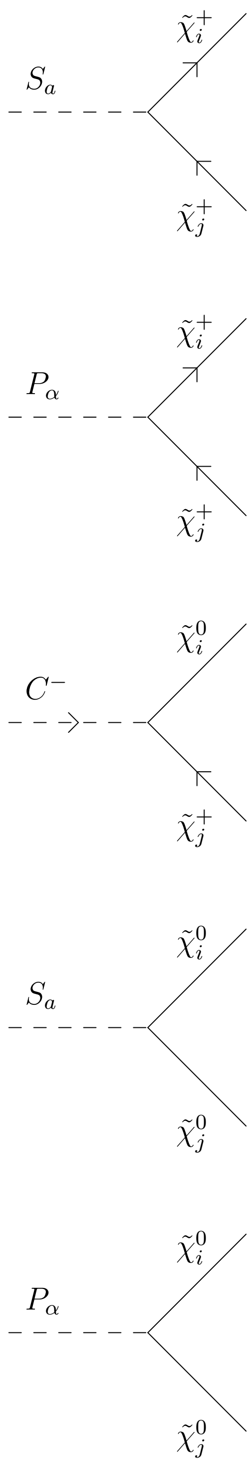



$$
\begin{aligned}
&-\tfrac{1}{4}ig^2(U^S_{a2}U^S_{b2}+U^S_{a1}U^S_{b1})\\
&-i\left(\tfrac{1}{4}g^2-\tfrac{\lambda^2}{2}\right)(U^S_{a1}U^S_{b2}+U^S_{a2}U^S_{b1})\sin 2\beta\\
&-\tfrac{1}{4}ig'^2(U^S_{a2}U^S_{b2}-U^S_{a1}U^S_{b1})\cos 2\beta\\
&-i\lambda(\lambda U^S_{a3}U^S_{b3}+kU^S_{a3}U^S_{b3}\sin 2\beta)
\end{aligned}
$$




$$
\begin{aligned}
&-\tfrac{1}{4}ig^2(U^P_{\alpha2}U^P_{\beta2}+U^P_{\alpha1}U^P_{\beta1})\\
&+i\left(\tfrac{1}{4}g^2-\tfrac{\lambda^2}{2}\right)(U^P_{\alpha1}U^P_{\beta2}+U^P_{\alpha2}U^P_{\beta1})\sin 2\beta\\
&-\tfrac{1}{4}ig'^2(U^P_{\alpha2}U^P_{\beta2}-U^P_{\alpha1}U^P_{\beta1})\cos 2\beta\\
&-i\lambda(\lambda U^P_{\alpha3}U^P_{\beta3}-kU^P_{\alpha3}U^P_{\beta3}\sin 2\beta)
\end{aligned}
$$

Figure 17 (continued): Feynman rules for the quartic self-interactions of the Higgs bosons.




$$-i(Q^*_{aij}P_L + Q_{aij}P_R)$$

$$R^*_{\alpha ij}P_L - R_{\alpha ij}P_R$$

$$i(Q'^{L*}_{ij}P_L + Q'^{R}_{ij}P_R)$$

$$-i(Q^{L''}_{aij}P_L + Q^{R''}_{aij}P_R)$$

$$R^{L''}_{\alpha ij}P_L + R^{R''}_{\alpha ij}P_R$$

Figure 18: Feynman rules for the couplings of Higgs bosons to neutralinos and charginos. The relevant factors $Q$ and $R$ are given in the text.

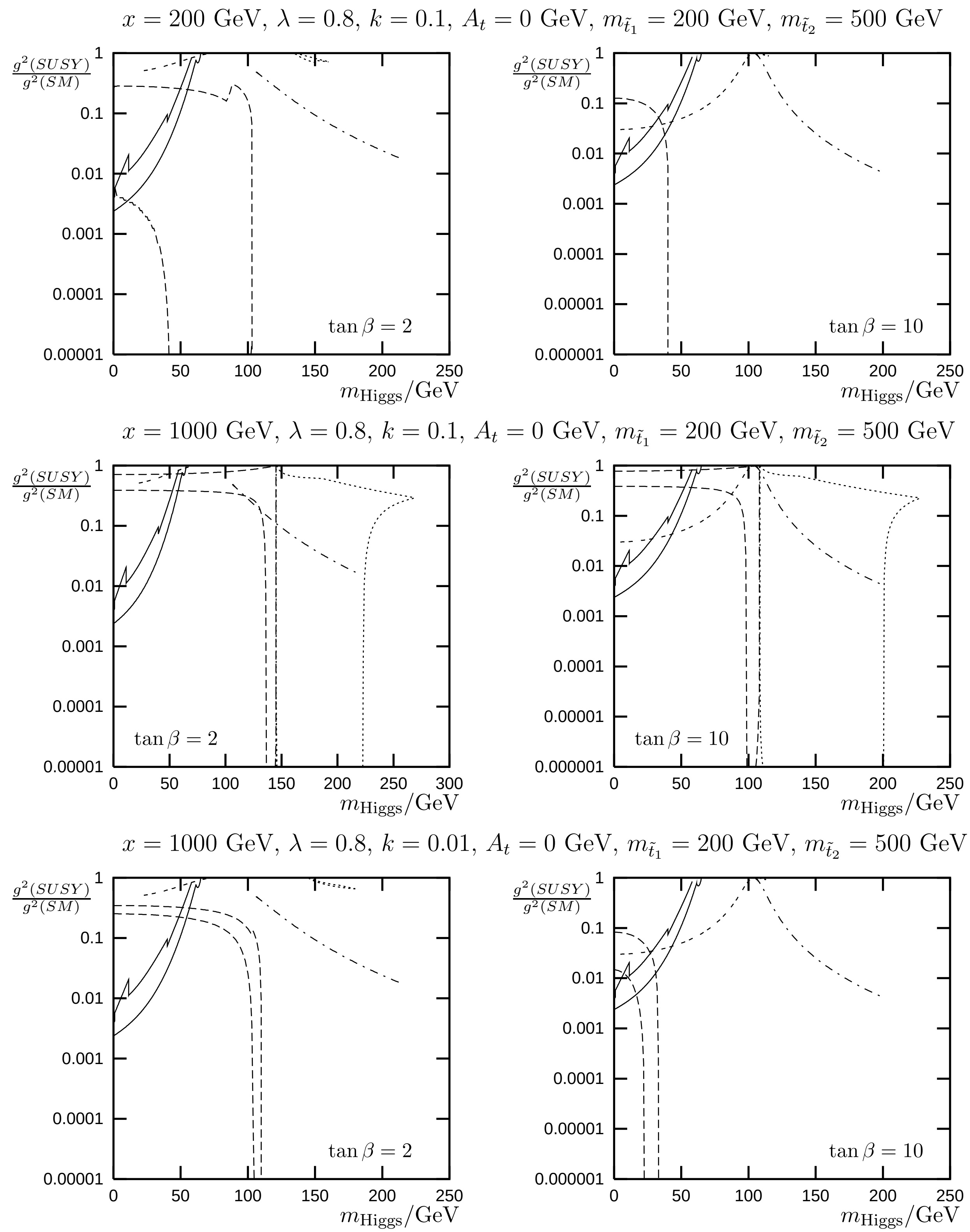



Figure 19: The ratios of the SUSY Higgs couplings to two $Z$ bosons relative to those of the SM. Shown are the range for the NMSSM couplings $g^2_{S_1ZZ}(NMSSM)/g^2_{\Phi ZZ}(SM)$ (dashed), $g^2_{S_2ZZ}(NMSSM)/g^2_{\Phi ZZ}(SM)$ (dotted), and the MSSM couplings $g^2_{hZZ}(MSSM)/g^2_{\Phi ZZ}(SM)$ (double dashed), $g^2_{HZZ}(MSSM)/g^2_{\Phi ZZ}(SM)$ (dashed-dotted). The solid lines denote the experimental bounds from LEP1 for a visibly and invisibly decaying Higgs boson.

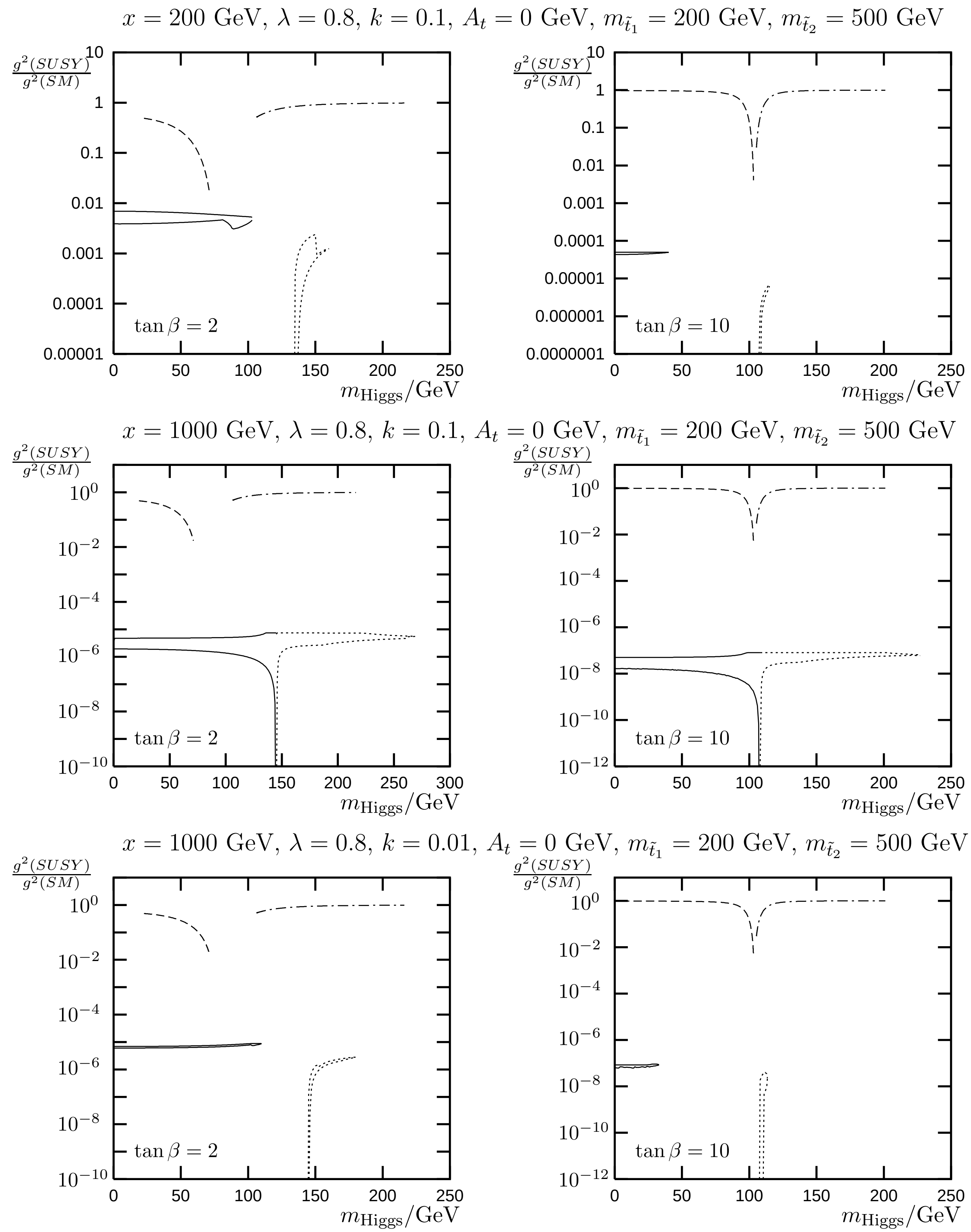



Figure 20: The squared couplings between a scalar and pseudoscalar Higgs boson and a $Z$ boson as a function of the scalar Higgs mass. Shown are the range for the NMSSM couplings $g^2_{S_1 P_1 Z}(NMSSM)/g^2_{\Phi ZZ}(SM)$ (solid line), $g^2_{S_2 P_1 Z}(NMSSM)/g^2_{\Phi ZZ}(SM)$ (dotted) and the MSSM couplings $g^2_{hAZ}(MSSM)/g^2_{\Phi ZZ}(SM)$ (dashed), $g^2_{HAZ}(MSSM)/g^2_{\Phi ZZ}(SM)$ (dashed dotted).

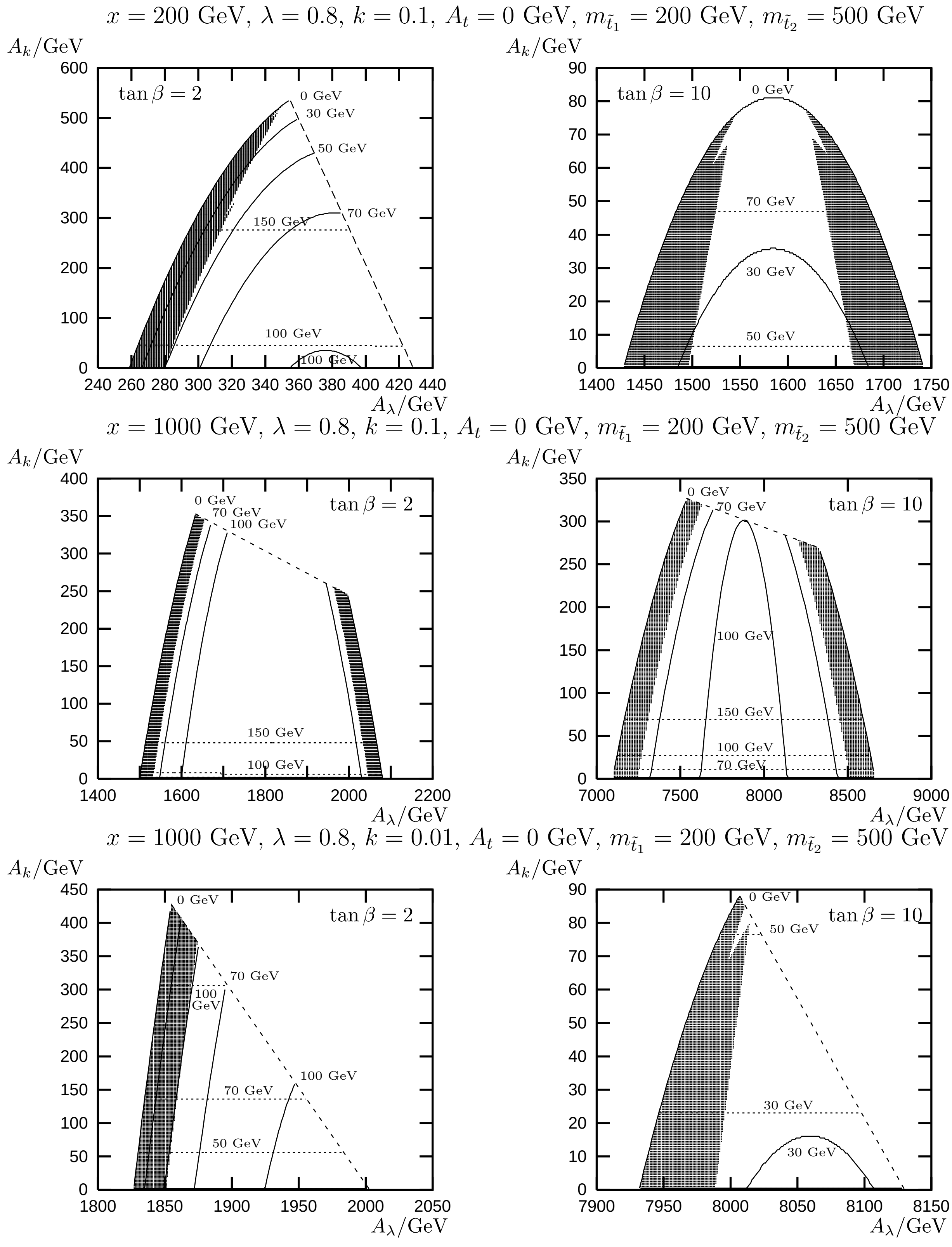



Figure 21: The experimentally excluded parameter space in the $(A_\lambda, A_k)$ plane (dark area). Also shown are the contour lines for the mass of the lightest Higgs scalar (solid) and of the light pseudoscalar Higgs boson (dotted). The region beyond the $m_{S_1} = 0$-GeV line and the dashed line is theoretically excluded as explained in the text.

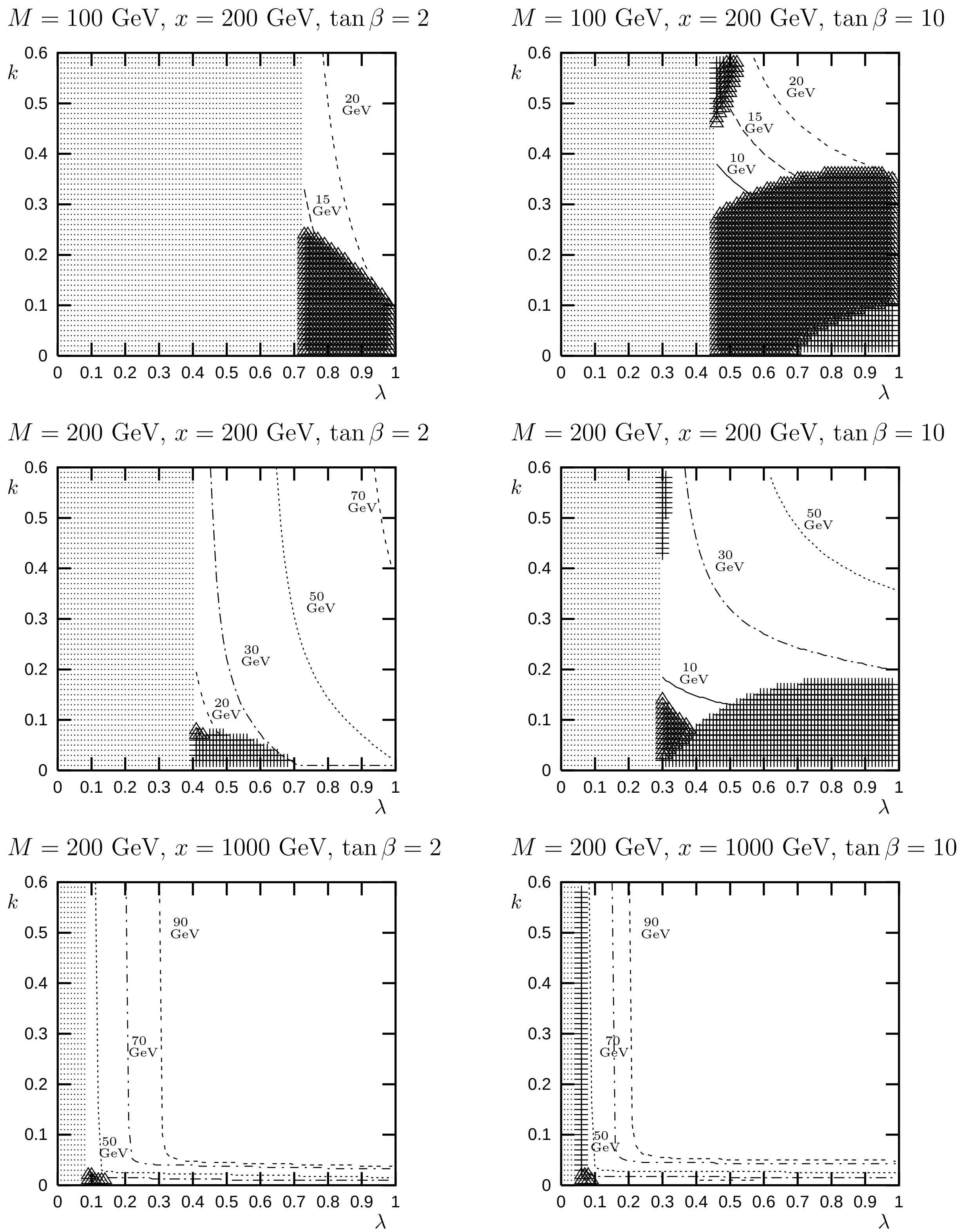



Figure 22: The excluded parameter space from neutralino search at LEP in the $(\lambda, k)$ plane for various values of $M$, $x$ and $\tan\beta$: from total $Z$ width measurements (dotted area), invisible $Z$ width measurements (checkered area) and direct neutralino search (dark area). Also shown are the mass contour lines of the lightest neutralino.

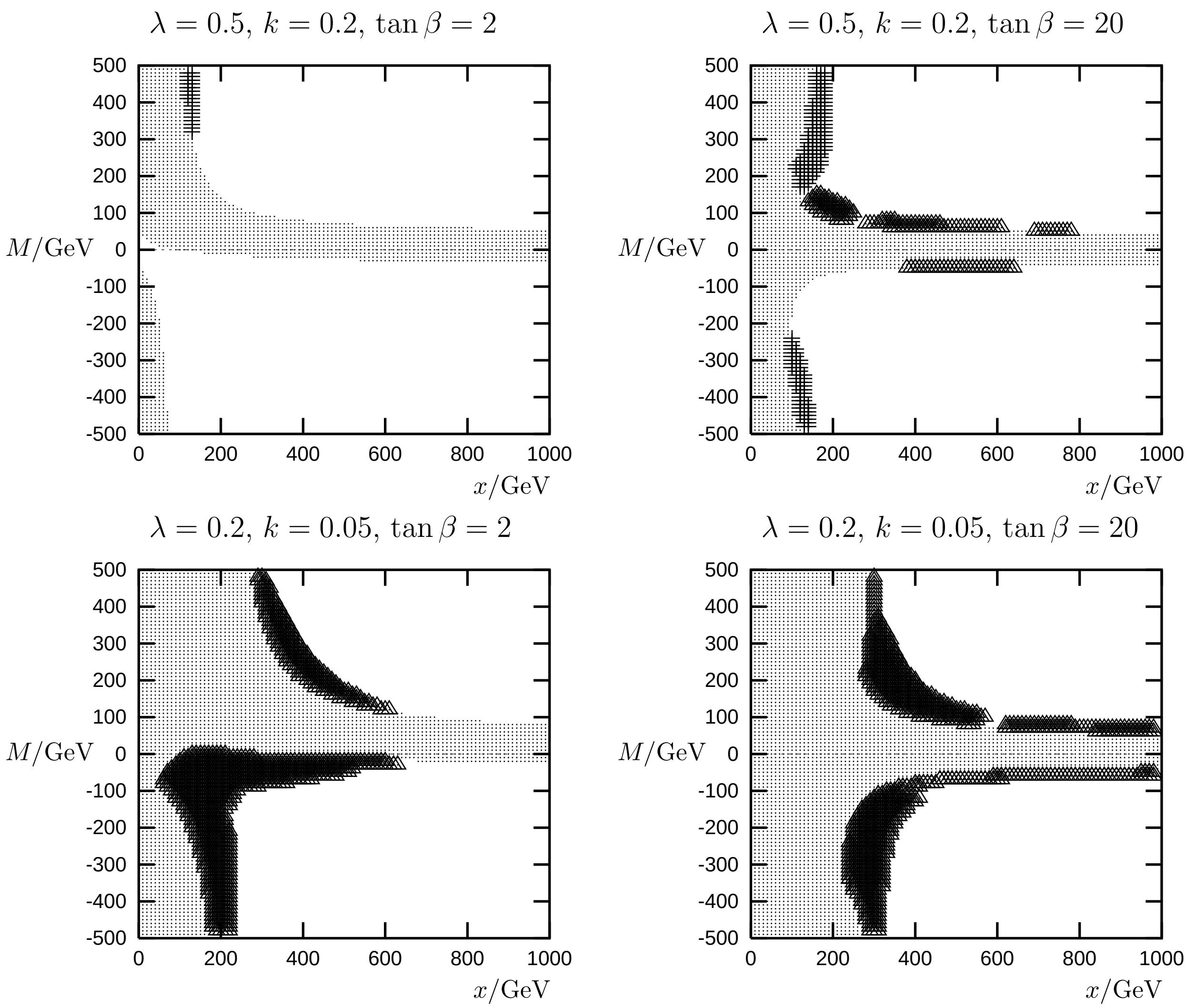



Figure 23: The excluded parameter space from neutralino search at LEP in the $(M, x)$ plane for various values of $\lambda$, $k$ and $\tan\beta$: from total $Z$ width measurements (dotted area), invisible $Z$ width measurements (checkered area) and direct neutralino search (dark area).

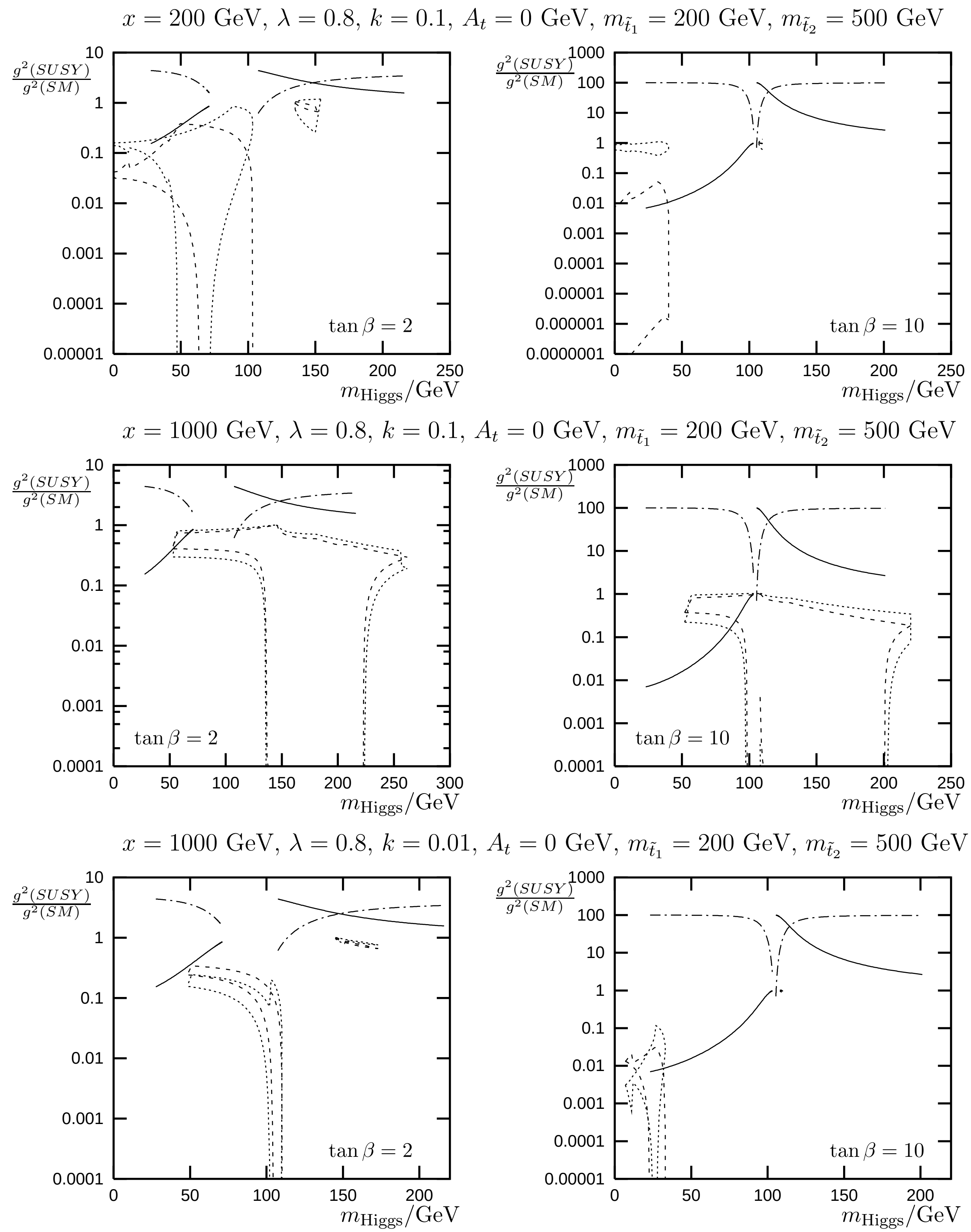



Figure 24: The squared ratios of the SUSY Higgs-quark-antiquark-couplings relative to the corresponding SM couplings. Shown are the MSSM couplings of the scalar Higgs bosons to top quarks (solid) and bottom quarks (dashed-dotted) as well as the range of the NMSSM couplings to top quarks (dashed) and bottom quarks (dotted).

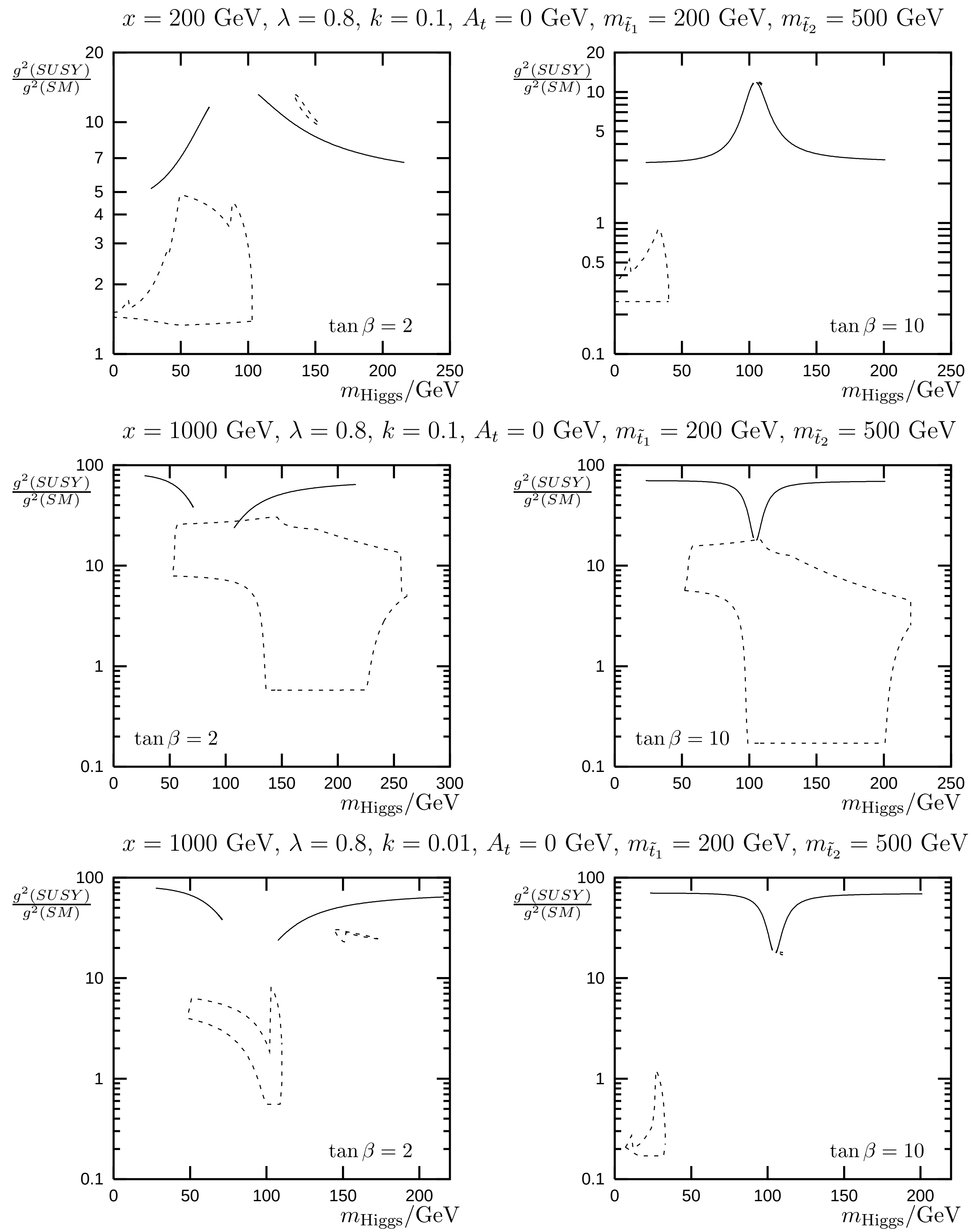



Figure 25: The SUSY Higgs couplings to two squarks as defined in eq. (155). Shown are the MSSM couplings (solid) as well as the range of the NMSSM couplings (dashed).

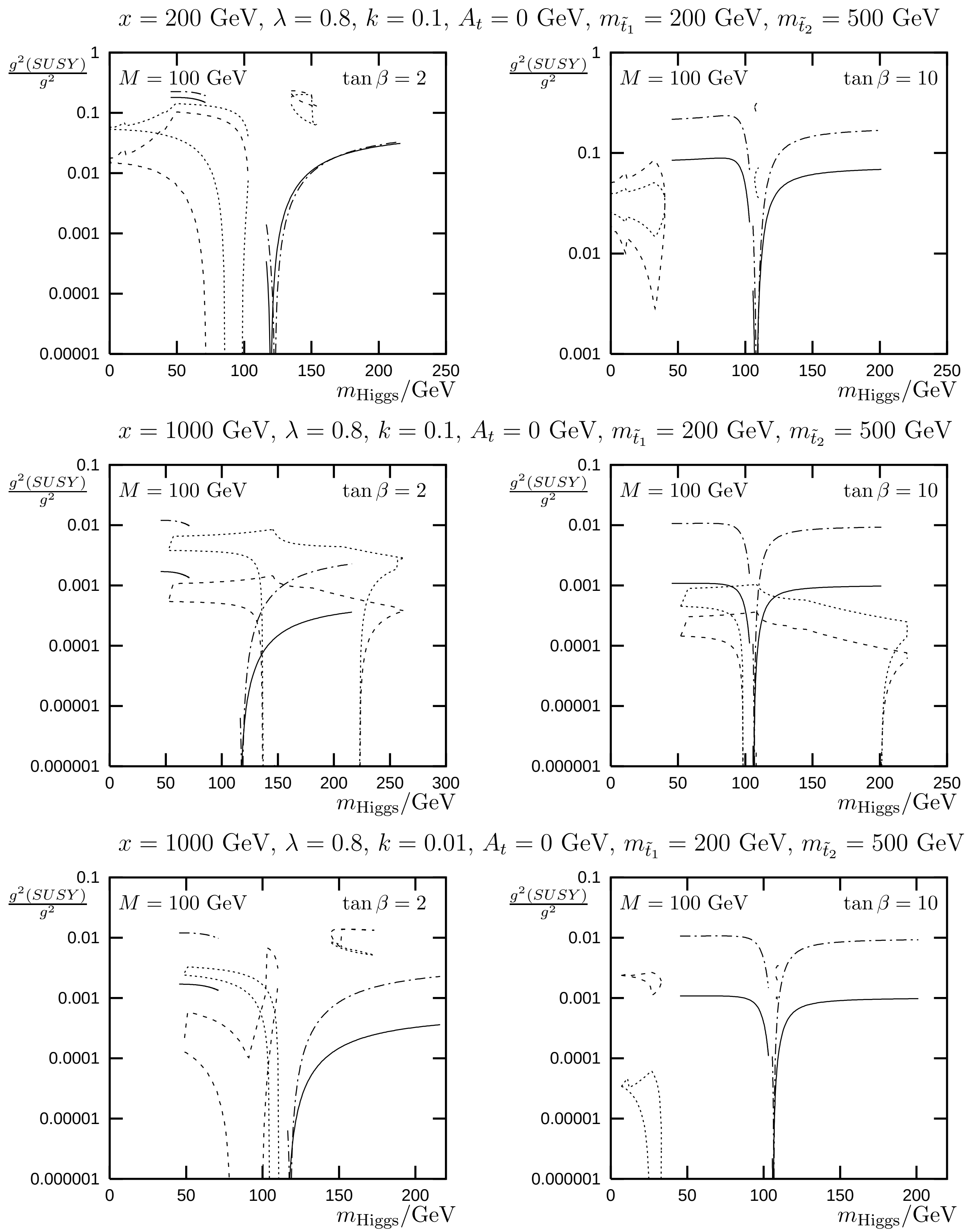



Figure 26: The squared SUSY Higgs couplings to neutralinos and charginos relative to $g^2$. Shown are the MSSM couplings of the scalar Higgs bosons to lightest neutralinos (solid) and light charginos (dashed-dotted) as well as the range of the NMSSM couplings to lightest neutralinos (dashed) and light charginos (dotted).

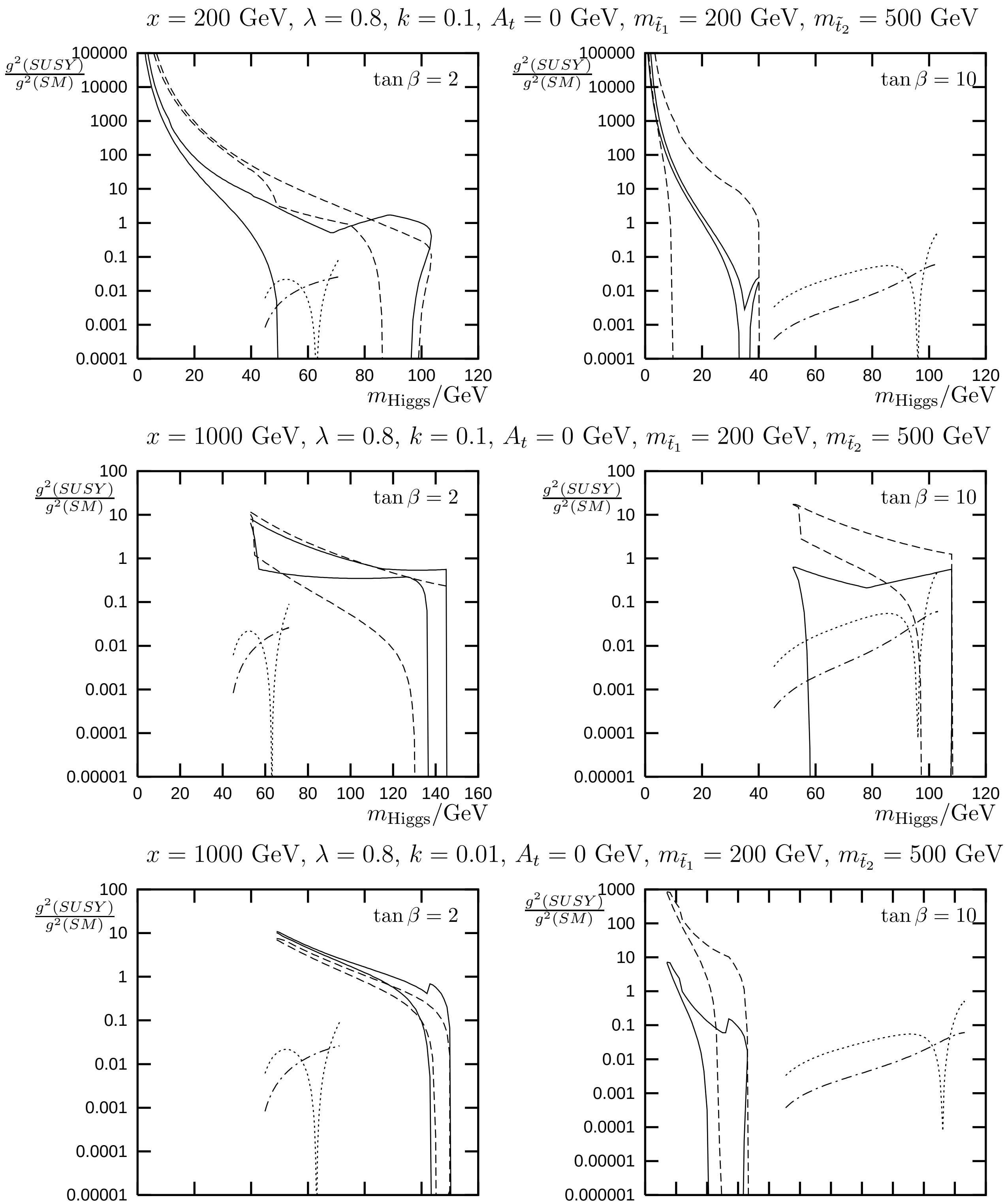



Figure 27: The squared ratios of the trilinear SUSY Higgs self-couplings relative to those of the SM. Shown are the range of the NMSSM couplings $g^2_{S_1S_1S_1}(NMSSM)/g^2_{\Phi\Phi\Phi}(SM)$ (solid), $g^2_{S_1P_1P_1}(NMSSM)/g^2_{\Phi\Phi\Phi}(SM)$ (dotted) and the MSSM couplings $g^2_{hhh}(MSSM)/g^2_{\Phi\Phi\Phi}(SM)$ (double dashed), $g^2_{hAA}(MSSM)/g^2_{\Phi\Phi\Phi}(SM)$ (dashed dotted).